1963

The Effects of a Specially Planned Mathematics Program on Pupil Achievement in Eighth Grade Mathematics.

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JOSEPH, Dorris George, 1922–
THE EFFECTS OF A SPECIALLY PLANNED MATHEMATICS PROGRAM ON PUPIL ACHIEVEMENT IN EIGHTH GRADE MATHEMATICS.

Louisiana State University, Ed.D., 1963
Education, general

THE EFFECTS OF A SPECIALLY PLANNED MATHEMATICS PROGRAM ON PUPIL ACHIEVEMENT IN EIGHTH GRADE MATHEMATICS

A Dissertation

Submitted to the Graduate Faculty of the
Louisiana State University and
Agricultural and Mechanical College
in partial fulfillment of the
requirements for the degree of
Doctor of Education

in

The Department of Education

by
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B.A., University of Southwestern Louisiana, 1951
M.Ed., Louisiana State University, 1953
August, 1963

ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation to his major professor, Dr. Thomas R. Landry and to Dr. John A. Hunter, who have given aid, assistance, and guidance throughout this study.

For their assistance and encouragement, the writer is grateful to the members of his committee, Dr. Donald E. Shipp, Jr., Dr. J. B. Kelley, Dr. L. L. Fulmer, Dr. Sam Adams, and Dr. Edwin A. Davis.

A debt of appreciation is due the administrators of St. Landry Parish who helped to make the study possible and to the twenty-three teachers of St. Landry Parish who gave of their time to participate in this study.

The writer is deeply grateful to his wife and three children for their help, understanding, and patience during the period of his graduate work.

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ABSTRACT

This study was concerned with the effects of a specially planned mathematics program on pupil achievement in eighth grade mathematics. This special program combined the traditional course with content carefully selected from the "new" mathematics.

The eighth grade teachers and pupils from twelve white public schools in St. Landry Parish, Louisiana, participated in the study. Teachers were paired and grouped on the basis of degrees held, years of teaching experience, years of teaching eighth grade mathematics, and training in the "new" mathematics. Students were grouped into four quartiles on the basis of the SRA Achievement Test in Arithmetic, Form A, administered in September.

Meetings were held with the teachers in the experimental group on the first Monday of each month from October through April. During these meetings the materials in the lesson plans were discussed. Each time the teachers were reminded to use only the normal class period for teaching purposes and were helped to supplement the material in the textbook with the new content planned for that month.

On the second Monday of each month meetings were held with the control group. These meetings consisted of discussions of the textual materials which were to be taught during the coming month. Teachers were instructed on placement of emphasis

on meaning and understanding and were instructed to use only the subject matter content found in their textbooks.

In April, 1963, Form B of the SRA Achievement Test in Arithmetic was administered to all the students who participated in the study. The analysis of variance was applied to the differences between means on Forms A and B to determine the significance of these differences which existed between the experimental and control groups.

In sixteen of the eighteen comparisons, the control group made greater gains; in two areas the experimental group made greater gains. When the F-test was applied, only four of the differences were found to be significant at the .05 level. All significant differences favored the control group. They were: (1) Arithmetic Concepts, Quartile III; (2) Arithmetic Computation, Quartile I; (3) Total Raw Scores by Quartiles, Quartile I; and (4) Total Score for the entire experimental and control groups.

Further examination revealed differences in the effects of the program on the lowest and highest ability groups. In the lowest ability group (Quartile I), the students who participated in the program planned for the control group did consistently better in the four areas tested than the students who participated in the program planned for the experimental group. Two of these areas were significant at the .05 level. In the highest ability group (Quartile IV), the students in the experimental group did slightly better or about as well as the control group in the four areas tested.

The following conclusions seem justified on the basis of the data gathered during the study:

1. The achievement of the control group was higher when measured by a conventional test than the achievement of the experimental group.
2. The program used with the control group seemed better adapted to the needs of the students in the three lower quartiles.
3. The smaller differences in Quartile IV indicated that the experimental program seemed better adapted to the higher ability group than the three lower groups.
4. Further experimentation with a larger sample, over a longer period of time, and with a variety of evaluative procedures is needed to determine the effect of the "new" mathematics content on student achievement at all ability levels.

CHAPTER I

INTRODUCTION

Recent discoveries in science and space explorations have caused the American public to become more aware of the importance of mathematics in daily living. This increased awareness is reflected in the criticisms of many lay and professional people concerning the quality of mathematics being presented to the youths in the schools. Implied in these criticisms is a charge that the schools are not meeting the needs of youth in this age of automation.

A major step was taken during the fall of 1960 to help the schools better meet their responsibilities when the National Council of Teachers of Mathematics, with the financial support of the National Science Foundation, conducted eight regional orientation conferences in mathematics in various parts of the United States. The purpose of these conferences was to give school administrators and mathematics supervisors information that would enable them to provide leadership in establishing new and improved mathematics.

As a result of these conferences many groups that were already seeking to develop better mathematics programs such as the University of Illinois Committee on School Mathematics, the School Mathematics Study Group, the Ball State Teachers College

Group, and the Maryland Mathematics Project, began studies which eventually led to the formulation of courses of study from the junior high school grades through the senior high school grades. These are the types of courses which have found their way into some of the schools in Louisiana and have caused much concern on the part of teachers, administrators, and parents as to the adaptability of these programs on the local level.

It is because of this concern for up-dating the mathematics program that the writer decided to pursue the present study in the hope that an acceptable program may be formulated to meet the needs of youth in the schools of Louisiana.

The Problem

The problem was to determine the effects of a specially planned mathematics program on pupil achievement in eighth grade mathematics. The experimental program combined the traditional course with content carefully selected from the "new" or "modern" mathematics.

Importance of the Study

Revision of the mathematics program has been a much discussed topic among educators recently. The chief concerns of this group seem to center around the following needs: (1) a change in the mathematics curriculum; (2) a change in the approach teachers and administrators should utilize; (3) an up-grading of the mathematics program, especially at the junior high school level;

and (4) a revision of the textbooks in mathematics which will serve as a guide to teachers and administrators in meeting the needs of the students of today.

The problem of meeting these needs involves curriculum building, and Hartung points out the complexity of the task as follows:

> . . . A program for teaching arithmetic is more than a set of things it might be interesting or desirable to try, or a collection of drill materials. A program must deal with the whole problem of teaching arithmetic to children. It must provide for a new and deeper understanding of arithmetic, and it must include adequate practice once these understandings have begun to develop. It must provide adequately for all pupils in the school, not just the fast learners or the remedial cases. It must be suitable for use, with minor adaptations, in many different school systems.[1]

Keeping these things in mind, the writer attempted through the development of a series of lesson plans that supplement a traditional course to organize a program to meet the needs of all the students in a heterogeneously grouped class. A test of this experimental program should give some direction in organizing more worthwhile mathematics programs for the junior high school grades.

Delimitation of the Problem

This study was limited to twenty-six sections of eighth grade mathematics in twelve white public schools of St. Landry Parish, Louisiana for the 1962-63 school year.

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[1]Maurice Hartung, H. Van Engen, and others, *Charting the Course for Arithmetic* (Dallas: Scott, Foresman and Company, 1960), p. 5.

Definitions of Terms Used

"New or Modern" Mathematics. The terms "new" and/or "modern" refer to the programs which have emerged from recent studies conducted in eighth grade mathematics such as the School Mathematics Study Group, The University of Maryland Mathematics Project, the Ball State Teachers College Group, and others.

Group A. Group A refers to those teachers and pupils who followed a regular program in mathematics using the materials from the textbook and supplemented by a series of units on the new mathematics.

Group B. Group B refers to those teachers and students who followed a regular program in mathematics using the materials from the textbook.

Special Program in Mathematics. The term "special program in mathematics" refers to a series of units in eighth grade mathematics which were constructed by the writer from materials drawn from many studies in the new mathematics. These units contained a list of objectives to be achieved, the materials needed for teaching the unit, possible approaches to the unit, an outline of subject-matter, suggested activities, and culmination procedures.

Procedures Used in Setting Up the Study

In April of 1962, the writer met with Superintendent Bergeron of St. Landry Parish, Louisiana and his staff. At that time the proposed study was presented and permission was given for the writer to meet with the principals of the white public schools of St. Landry Parish.

The writer met with the principals during the first week of May, 1962. At this meeting the proposed study was presented and in response, the principals agreed to participate if their teachers accepted the responsibility for carrying out the study. Subsequently, letters were sent to all eighth grade teachers in the parish explaining the nature of the study. A questionnaire was attached for the purpose of securing such information as degree held, teaching experience, and mathematics teaching experience. (Copies of the letter and the questionnaire may be found on pages 200 and 201 of the Appendix.) The principals were instructed to ask their teachers to return the questionnaire to the writer if they desired to participate in the study. The response was gratifying as all of the eighth grade teachers--twenty-three representing twenty-six sections of eighth grade mathematics--responded favorably. In the twenty-six sections, there were six hundred ten students.

Procedures which were used in carrying out the study will be found in Chapter II.

Survey of the Related Literature

No studies were located that supplemented the traditional program being used in 1962-63 in the eighth grade in St. Landry Parish schools with elements of the "new" or "modern" mathematics.

Many studies, however, were located that had some similarities to the present study. For example, studies have been made which relate to the preparation of elementary teachers for teaching mathematics.[2] Studies have also been made that test in-service procedures likely to promote growth in arithmetic.[3] A third example are those studies that relate to the effect of meaningfulness on learning.[4] Still others are concerned with special teaching methods.[5] All of the examples contain elements that related to the present study but none of them tested the same variable which this study tested.

Since the St. Landry study was designed to test the extent to which the introduction of content from the "new" or "modern" mathematics into a traditional program would bring about more understanding and consequently more progress, the studies related to increasing understanding are of some interest. In recent years and particularly since 1928, a number of studies have been carried out for the purpose of determining the effect of meaningful teaching

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[2]M. Vere DeVault, "An In-Service Mathematics Program for Intermediate Grade Teachers," The Arithmetic Teacher, VIII (February, 1961), p. 65.

[3]Ronie E. Rudd, "Enrichment for the Talented in Arithmetic," The Arithmetic Teacher, VIII (March, 1961), pp. 135-37.

[4]Russell E. Alkire, "An Experimental Study of the Value of a Meaningful Approach to the Operation of Division with Common Fractions," (Unpublished Master's thesis, Claremont College, 1949).

[5]Charles J. Faulk, "The Effect of the Use of a Particular Method on Achievement in Problem Solving in Sixth Grade Arithmetic," (Unpublished Doctoral dissertation, Louisiana State University, 1960).

of arithmetic on learning. Such studies as those conducted by Alkire,[6] Anderson,[7] Brownell and Moser,[8] and Howard[9] have shown that meaningful teaching results in learning that is superior to that achieved under other kinds of teaching.[10] None of these studies, however, used "new" mathematics content in order to improve understanding.

In recent years, much has been done in the area of the "new" or "modern" mathematics and although the recency of the new program makes a complete evaluation impossible, many of its advocates make extensive claims for it. Among these claims is the modest contention that pupils in the "new" program do as well as pupils pursuing a traditional program when measured by conventional tests. Even though this study did not introduce an entirely new program, it provided an opportunity to test this contention when applied to a traditional program supplemented by selected aspects of the "new" mathematics.

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[6] Alkire, *loc. cit.*

[7] Lester G. Anderson, "Quantitative Thinking as Developed under Connectionist and Field Theories of Learning," *Learning Theory in School Situations* (Minneapolis: University of Minnesota Press, 1949), pp. 40-73.

[8] William A. Brownell and Harold E. Moser, "Meaningful Versus Mechanical Learning," *Duke University Research Studies in Education*, VIII (Durham: Duke University Press, 1949), p. 207.

[9] Charles F. Howard, "Three Methods of Teaching Arithmetic," *California Journal of Educational Research*, I (January, 1950), pp. 3-7.

[10] Vincent J. Glennon and C. W. Hunnicutt, "What Does Research Say about Arithmetic?" A Report Prepared for the Association for Supervision and Curriculum Development. Washington: National Education Association, 1952. p. 12.

CHAPTER II

PROCEDURES USED IN THE STUDY

The procedure used in initiating and conducting the study are grouped under the following topics for presentation: (1) selecting the schools, (2) equating the groups, (3) planning with the experimental groups, (4) planning with the control group, and (5) determining pupil achievement.

Selecting the Schools

Twelve white, public schools of St. Landry Parish, Louisiana, participated in the study. The twelve schools representing both the urban and rural districts were: (1) Lawtell High School, (2) Eunice Elementary School, (3) Market Street Elementary School, Opelousas, (4) Paul Pavy Elementary School, Opelousas, (5) Port Barre High School, (6) Krotz Springs Junior High School, (7) Melville High School, (8) Palmetto High School, (9) Washington High School, (10) Grand Prairie High School, (11) Arnaudville High School, and (12) Sunset High School.

Equating the Groups

Twenty-three teachers were paired and grouped. Their pupils were designated as control and experimental groups.

Pairing and grouping the teachers. Teachers in the twelve schools were paired on the basis of the information they supplied on data sheets which they completed at the beginning of the study. As a first step, the items of information on each teacher were recorded on small cards. Then, the cards were matched as closely as possible using the following data: (1) degree held, (2) years teaching eighth grade mathematics, (3) total years teaching experience, and (4) in-service or other teaching in the new mathematics. After the cards were matched, the flip of a coin was used to assign teachers to the control and experimental groups. Since there were twenty-three teachers, one could not be paired, so the flip of the coin was used to assign him to the control group.

Table I gives comparative data for the two groups of teachers. It shows that there were five teachers in each group with special training in the "new" mathematics and that both groups had approximately the same number of years of experience teaching eighth grade mathematics. It shows also that the experimental group was slightly favored in the area of academic training while the control was favored in total years of teaching experience.

Grouping the pupils. The pupils in the rooms of the experimental teachers became the experimental group of pupils, and those in the rooms of the control teachers became the control group. No attempts were made to pair individual pupils. Instead, the pupils in each group were divided into four sections (or quartiles) on the basis of an achievement test, the SRA Achievement

TABLE I

DEGREES HELD, TEACHING EXPERIENCE, AND SPECIAL TRAINING OF PAIRED TEACHERS

EXPERIMENTAL					CONTROL				
Teacher Number	Degree	Number of Years Experience	Number of Yrs. Eighth Grade Exp.	Special Training in New Math.	Teacher Number	Degree	Number of Years Experience	Number of Yrs. Eighth Grade Exp.	Special Training in New Math.
9	B.A.	2	1	Yes	13	B.A.	2	1	No
4	B.A.	4	2	Yes	19	B.A.	5	1	No
8	B.S.	5	3	No	21	B.A.	5	5	Yes
2	B.A.	6	2	No	14	M.Ed.	8	6	No
6	M.Ed.	7	7	Yes	17	B.A.	7	7	Yes
11	B.S.	7	3	Yes	12	B.A.	9	9	Yes
5	M.Ed.	8	8	Yes	20	M.Ed.	10	6	Yes
1	M.Ed.	9	2	No	15	B.S.	17	10	No
3	B.A.	20	2	No	16	B.S.	20	1	Yes
7	M.Ed.	20	2	No	18	B.S.	20	3	No
10	3 yr.	23	13	No	22	B.A.	25	14	No
					23	B.A.	25	2	No

Test in Arithmetic, Form A, given in September, 1962. Those students in each group whose total raw scores ranged from twenty-six through fifty-one were placed in Quartile I. Those students in each group whose total raw scores ranged from fifty-two through sixty-four were placed in Quartile II. Those students in each group whose total raw scores ranged from sixty-five through seventy-eight were placed in Quartile III. Those students in each group whose total raw scores ranged from seventy-nine through one hundred twenty-five were placed in Quartile IV.

Tables II, III, IV, and V show comparisons of the raw scores made by the pupils in the experimental and control groups in arithmetic reasoning, arithmetic concepts, arithmetic computation, and total raw scores by quartiles at the beginning of the study. Table II shows the raw scores for the two groups of students who were placed in Quartile I. Tables III, IV, and V show these data for the students in Quartiles II, III, and IV, respectively.

The differences between the mean raw scores were computed for each quartile. A difference of .4 was found for Quartile I, .3 for Quartile II, .6 for Quartile III, and 1.6 for Quartile IV. None of these differences were statistically significant, so the experimental and control groups in each quartile were considered equal.

TABLE II

DATA SHOWING RAW SCORES OF PUPILS IN QUARTILE I OF THE EXPERIMENTAL AND CONTROL GROUPS FOR ARITHMETIC REASONING (A), ARITHMETIC CONCEPT (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES AS DETERMINED BY THE SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, IN SEPTEMBER, 1962

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
8	10	9	27	9	9	8	26
9	10	10	29	9	9	9	27
10	6	14	30	8	6	17	31
7	11	14	32	12	12	8	32
14	12	7	33	12	8	12	32
9	5	20	34	15	6	12	33
12	8	14	34	6	16	13	35
9	13	12	34	10	11	14	35
14	7	14	35	10	15	10	35
10	15	10	35	14	10	12	36
12	9	14	35	11	10	16	37
9	8	19	36	14	6	17	37
9	10	17	36	15	11	11	37
14	10	13	37	16	12	9	37
8	7	23	38	9	11	18	38
10	15	13	38	15	14	9	38
10	6	22	38	11	16	11	38
10	10	18	38	15	11	13	39
13	5	20	38	15	10	14	39
11	9	18	38	9	15	16	40
8	15	17	40	14	14	12	40
8	11	22	41	20	8	12	40
11	10	20	41	21	6	13	40
17	11	13	41	10	10	20	40
21	8	13	42	12	13	18	43

TABLE II (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
16	9	17	42	12	13	18	43
13	11	18	42	11	7	25	43
13	10	19	42	13	11	20	44
16	10	17	43	16	5	23	44
11	10	22	43	14	15	16	45
15	12	17	44	18	12	16	46
15	12	17	44	11	15	20	46
23	14	7	44	18	12	16	46
17	11	16	44	15	15	16	46
15	17	12	44	13	7	26	46
16	14	14	44	12	12	22	46
12	10	23	45	17	6	23	46
14	13	18	45	12	12	22	46
15	12	18	45	10	19	18	47
17	14	14	45	15	14	18	47
15	11	19	45	14	13	20	47
22	9	14	45	18	21	19	48
18	12	15	45	26	7	15	48
19	10	16	45	15	15	18	48
15	16	14	45	16	17	16	49
14	11	21	46	15	11	23	49
16	9	21	46	16	12	21	49
15	14	17	46	26	14	9	49
19	11	17	47	16	16	17	49
19	10	18	47	13	10	26	49
20	9	18	47	21	16	12	49
14	9	24	47	20	14	15	49
19	12	16	47	14	11	24	49

TABLE II (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
15	12	20	47	9	14	26	49
15	12	20	47	17	15	18	50
18	10	20	48	8	10	32	50
17	14	17	48	21	10	19	50
15	8	25	48	21	12	17	50
17	10	21	48	24	7	19	50
16	13	19	48	18	13	19	50
19	13	16	48	16	17	17	50
24	15	9	48	20	12	18	50
21	9	19	49	13	12	26	51
18	11	20	49	14	14	23	51
18	8	23	49	19	16	16	51
17	13	19	49	13	9	29	51
12	10	27	49	21	13	17	51
14	16	19	49	25	17	9	51
12	14	23	49	13	15	23	51
14	9	27	50				
15	12	23	50				
16	10	24	50				
17	20	23	50				
17	11	22	50				
20	9	22	51				
19	10	22	51				
19	10	22	51				
22	15	14	51				
15	11	25	51				
24	12	15	51				

TABLE III

DATA SHOWING RAW SCORES OF PUPILS IN QUARTILE II OF THE EXPERIMENTAL AND CONTROL GROUPS FOR ARITHMETIC REASONING (A), ARITHMETIC CONCEPT (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES AS DETERMINED BY THE SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, IN SEPTEMBER, 1962

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
18	15	19	52	27	16	9	52
19	13	20	52	14	14	24	52
21	10	21	52	26	10	16	52
15	12	25	52	15	23	14	52
16	14	22	52	18	12	23	52
19	13	20	52	18	7	27	52
13	11	28	52	16	16	20	52
18	16	19	53	22	16	15	53
24	14	15	53	21	12	20	53
17	11	26	54	16	13	24	53
19	16	19	54	23	9	21	53
19	14	21	54	21	15	18	54
17	17	20	54	12	14	28	54
13	10	32	55	22	11	21	54
18	17	20	55	17	8	29	54
17	13	25	55	27	14	14	55
13	17	25	55	16	19	20	55
22	17	16	55	16	14	23	55
14	15	26	55	17	15	23	55
17	12	26	55	20	13	22	55
24	15	16	55	20	14	21	55
16	14	25	55	20	13	23	56
14	13	29	56	15	16	25	56
18	12	26	56	18	13	25	56
17	14	25	56	20	9	27	56

TABLE III (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
19	10	27	56	22	14	20	56
15	13	28	56	16	16	24	56
28	10	18	56	19	14	23	56
21	15	20	56	19	9	28	56
17	17	23	57	12	21	23	56
15	15	27	57	24	12	21	57
19	13	25	57	18	14	25	57
14	15	28	57	25	14	18	57
22	16	19	57	18	10	29	57
18	14	25	57	18	19	20	57
20	10	28	58	22	17	18	57
10	14	34	58	23	12	23	58
19	13	26	58	20	14	24	58
19	15	24	58	25	19	14	58
17	15	26	58	23	13	23	59
33	11	15	59	17	21	21	59
19	14	26	59	28	14	17	59
18	16	25	59	26	19	14	59
13	11	35	59	25	11	23	59
14	18	27	59	22	12	25	59
23	8	28	59	17	18	25	60
20	17	22	59	17	16	27	60
26	13	20	59	22	15	23	60
18	11	30	59	21	20	19	60
22	15	23	60	25	18	17	60
18	17	25	60	21	18	21	60
0	22	38	60	23	15	22	60
15	17	28	60	15	18	28	61
21	17	23	61	22	18	21	61

TABLE III (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
20	15	26	61	27	11	23	61
24	13	24	61	20	20	21	61
20	12	29	61	19	18	25	62
20	16	26	62	16	22	24	62
19	15	28	62	14	15	33	62
25	19	18	62	12	19	31	62
24	17	21	62	18	19	25	62
20	21	21	62	15	15	32	62
26	13	23	62	18	16	28	62
17	13	31	62	14	6	42	62
23	12	27	62	24	13	25	62
17	15	31	63	21	16	25	62
22	22	19	63	19	12	31	62
19	14	31	64	22	19	22	63
24	14	26	64	16	18	29	63
20	16	28	64	19	13	31	63
16	21	27	64	23	14	26	63
22	20	22	64	23	14	27	64
20	17	27	64	20	17	27	64
26	17	21	64	18	15	31	64
				20	14	30	64
				20	19	25	64

TABLE IV
DATA SHOWING RAW SCORES OF PUPILS IN QUARTILE III OF THE EXPERIMENTAL AND CONTROL GROUPS FOR ARITHMETIC REASONING (A), ARITHMETIC CONCEPT (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES AS DETERMINED BY THE SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, IN SEPTEMBER, 1962

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
17	13	35	65	28	17	20	65
23	16	26	65	18	23	24	65
17	17	31	65	24	13	28	65
23	16	26	65	21	14	30	65
26	16	23	65	30	17	18	65
22	18	25	65	29	19	18	66
17	16	33	66	24	12	30	66
22	21	23	66	19	20	27	66
21	19	26	66	19	22	25	66
28	18	20	66	25	15	26	66
19	12	36	67	20	20	26	66
25	14	28	67	20	17	29	66
16	21	30	67	29	16	21	66
13	21	33	67	17	18	31	66
21	13	33	67	25	14	27	66
20	14	34	68	25	17	34	66
22	15	31	68	23	14	30	67
26	15	27	68	23	14	30	67
28	15	26	69	25	21	21	67
29	13	27	69	27	13	27	67
27	15	27	69	21	13	33	67
21	19	29	69	24	18	25	67
25	16	28	69	22	13	32	67
30	18	21	69	15	16	36	67
26	13	31	70	24	13	31	68
24	19	27	70	27	14	27	68

TABLE IV (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
25	20	25	70	24	16	28	68
25	14	32	71	24	13	32	69
22	17	32	71	20	21	28	69
25	21	25	71	18	12	39	69
21	21	29	71	22	16	31	69
24	19	28	71	26	14	30	70
25	17	29	71	24	14	32	70
26	24	31	71	29	13	28	70
25	22	25	72	26	19	26	71
24	22	26	72	23	20	28	71
22	19	31	72	31	14	27	72
21	16	35	72	26	23	23	72
30	16	27	73	33	12	27	72
26	18	29	73	27	20	25	72
25	15	33	73	21	18	34	73
20	19	34	73	27	19	27	73
24	17	32	73	21	13	39	73
23	12	38	73	26	18	29	73
23	17	33	73	30	19	24	73
30	21	23	74	28	21	25	74
24	20	30	74	25	25	24	74
25	20	29	74	23	18	33	74
34	13	27	74	33	19	22	74
25	17	32	74	28	12	35	74
17	18	39	74	22	19	34	75
23	15	36	74	25	15	35	75
26	18	30	74	25	23	27	75
27	21	26	74	24	20	31	75
35	22	18	75	31	17	27	75

TABLE IV (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
31	14	30	75	21	17	37	75
21	16	38	75	25	21	29	75
28	17	30	75	27	16	33	76
27	14	34	75	25	19	32	76
21	17	37	75	21	21	34	76
21	17	37	75	29	18	29	76
22	17	36	75	25	15	36	76
26	19	30	75	34	21	21	76
34	15	26	75	28	13	35	76
24	23	29	76	23	19	34	76
31	16	29	76	26	17	34	77
23	14	39	76	31	18	28	77
27	21	29	76	21	19	37	77
22	22	33	77	18	24	35	77
28	14	35	77	30	20	27	77
26	17	34	77	28	18	32	78
25	20	32	77	28	17	33	78
28	18	32	78	31	19	38	78
38	19	21	78	30	17	31	78
18	25	35	78	25	17	36	78
25	24	29	78				
28	19	31	78				

TABLE V

DATA SHOWING RAW SCORES OF PUPILS IN QUARTILE IV OF THE EXPERIMENTAL AND CONTROL GROUPS FOR ARITHMETIC REASONING (A), ARITHMETIC CONCEPT (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES AS DETERMINED BY THE SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, IN SEPTEMBER, 1962

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
31	15	33	79	25	24	30	79
24	21	34	79	35	15	29	79
37	13	29	79	30	18	32	80
23	22	34	79	30	22	38	80
30	14	35	79	27	21	32	80
28	19	32	79	28	21	31	80
29	19	31	79	25	21	34	80
29	15	36	80	28	19	33	80
36	11	33	80	30	19	31	80
36	11	33	80	32	15	33	80
22	27	31	80	26	18	37	81
27	24	29	80	30	22	29	81
31	17	33	81	31	16	24	81
26	16	39	81	25	24	33	82
30	16	35	81	28	19	35	82
27	21	34	82	30	19	33	82
25	19	38	82	31	13	38	82
24	25	33	82	24	17	41	82
28	25	29	82	23	24	35	82
32	17	33	82	32	23	27	82
31	17	34	82	28	21	33	82
32	18	32	82	28	22	32	82
27	20	36	83	28	18	37	83
28	20	35	83	26	21	36	83

TABLE V (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
27	27	29	83	24	19	40	83
24	23	36	83	26	19	39	84
30	23	30	83	27	19	38	84
26	22	35	83	29	17	38	84
26	20	38	84	30	18	36	84
27	19	38	84	33	21	30	84
23	26	35	84	33	20	31	84
27	21	37	85	31	16	37	84
30	21	34	85	35	18	32	85
31	22	32	85	30	21	35	86
26	22	37	85	32	23	31	86
29	23	34	86	31	20	35	86
27	20	39	86	29	22	35	86
35	20	31	86	39	24	24	87
36	16	35	87	32	17	38	87
30	21	36	87	28	23	37	88
27	21	40	88	30	20	38	88
35	20	33	88	31	19	40	90
30	22	36	88	37	24	29	90
26	27	36	89	36	19	35	90
29	21	39	89	35	24	32	91
31	17	41	89	38	20	33	91
31	20	38	89	30	29	32	91
30	21	40	91	32	19	40	91
26	25	42	93	25	26	40	91
30	23	41	94	29	18	45	92
37	23	34	94	26	29	37	92
27	28	40	95	37	25	30	92

TABLE V (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
31	20	44	95	33	20	40	93
34	22	39	95	35	16	42	93
29	24	43	96	30	22	42	94
32	26	38	96	31	20	43	94
35	22	40	97	37	17	40	94
32	22	44	98	31	25	38	94
33	26	40	99	35	22	38	95
32	23	44	99	29	25	41	95
39	23	38	100	38	22	35	95
33	28	40	101	37	20	39	96
28	32	41	101	34	24	39	97
37	24	40	101	34	24	39	97
30	29	43	102	41	22	35	98
32	29	41	102	37	21	40	98
38	24	40	102	29	28	41	98
41	28	36	105	39	24	37	100
33	25	47	105	34	22	44	100
34	29	43	106	40	20	41	101
41	24	43	108	32	28	41	101
41	27	41	109	37	23	42	102
37	36	39	112	33	27	42	102
43	30	44	117	47	28	50	102
				27	31	45	103
				41	25	40	106
				38	26	42	106
				40	28	39	107
				38	28	41	107
				33	32	43	108

TABLE V (Continued)

EXPERIMENTAL GROUP				CONTROL GROUP			
A	B	C	Total	A	B	C	Total
				38	27	43	108
				40	30	41	111
				38	34	40	112
				39	30	44	113
				48	33	44	125

<u>Planning with the Teachers in the Experimental Group</u>

The teachers in the experimental group met with the writer on the first Monday of every month from September through April, 1963. These meetings were held in the Paul Pavy Elementary School, Opelousas, Louisiana from 4:30 p.m. until 5:30 p.m.

<u>The September meeting</u>. The first meeting in September was used to explain the study and the responsibilities of each teacher in the group. The teachers were informed that they were expected to attend all meetings and to carry out the procedures for teaching their mathematics classes as outlined at each of these meetings. The regular eighth grade class mathematics textbook was to be used as the core of the mathematics program. It was explained that at each succeeding meeting each teacher would receive a mimeographed group of plans based on the so-called "modern mathematics." These plans were to be used as a supplement to the material covered in the textbook.

<u>Subsequent meetings</u>. At the meetings held in October, November, December, January, February, and March, each teacher was given a set of plans for use during the coming month. (These plans are included in Appendixes A through F.) At each meeting the plan of the coming month was distributed to the teachers and a short discussion period was conducted to clarify or answer any questions the teachers had concerning the plan. During these discussions the teachers were instructed to use the normal class time for teaching purposes and to incorporate the supplementary content included in

the plan with the material in the textbook. They were cautioned against giving more help than would ordinarily be given to the student. They were told that they could mimeograph the parts of the plan which would serve as practice material for the students and that they were free to use any aids which they would ordinarily have used. They were encouraged to allow time for the student to discuss the material rather fully before proceeding to the next phase.

In addition to the general instructions given to the experimental group of teachers, specific instructions and directions accompanied distribution of each new plan each month. These discussions focused on objectives, subject matter, and suggested activities. The objectives listed in the plan were discussed by the teachers and means of achieving these objectives were explained. The subject matter outline to be taught by all experimental teachers was next explained in detail. Finally, the activities suggested in the plan were reviewed.

During the discussions that followed, stress was placed on the teachers covering the subject matter content of the lesson plan because this was the one big difference between the procedures used by the experimental group and those used by the control group.

<u>A summary of one of the plans</u>. The material found on page nine of the textbook, <u>Making Sure of Arithmetic--Grade Eight</u>, and the material found in Appendix A, "Finding the Least Common Multiple," will illustrate how one of the plans was used.

On page nine of the textbook is a presentation entitled "How to Find the Common Denominator when Adding Fractions." The presentation begins with these two statements: (1) "before two or more fractions can be added, their denominators must be the same," and (2) "when all the fractions to be added have the same denominator this denominator is called the common denominator," and follows with examples that are expected to help the pupils better understand the mathematical operation involved. The authors continue to explain that if the denominators are unlike, the student may then take the larger of the two denominators to determine a common denominator, multiply it by two, and determine whether or not the other denominator is divisible into this product. If it is not possible, it is suggested that the student multiply the larger denominator by three, and so on, until he arrives at a denominator which is divisible by the smaller one.

In this particular instance, the experimental teachers were told to teach the material found in the lesson plan (Appendix A) before presenting the material found in the textbook discussed above. From the lesson plan provided the teachers on finding the least common multiple, they began by explaining the meaning of the terms "multiple," and "common multiples." They followed this introduction with a discussion of the difference between a factor and a multiple and a presentation of examples so as to re-enforce the child's understanding of the terms. Subsequently, the idea of least common multiples was developed. Concepts from previous plans were taught in order to show the relationships which exist between the various phases of mathematics. After a thorough discussion of the above material, the pupils were given practice work so as to insure individual understanding and to provide an opportunity for the teacher to assist those students who were in need of help. As a final step, the material from the textbook was related to the material in the lesson plan emphasizing multiples, least common multiples, factors, factoring, and common denominators.

Developing the material stated in the above summary was expected to take from one to three class periods.

Planning with the Teachers in the Control Group

The teachers in the control group met with the writer on the second Monday of every month from September through April, 1963. These meetings were also held in the Paul Pavy Elementary School, Opelousas, Louisiana from 4:30 p.m. to 5:30 p.m.

The September meeting. The first meeting in September was used to explain the study and the responsibilities of each teacher in the group. The teachers were cautioned to use normal class time in teaching arithmetic, to give no more individual attention than ordinarily would be given, to use the teaching aids which they would ordinarily have used, and to teach the subject matter contained in the textbook.

Subsequent meetings. At the meetings held in October, November, December, January, February, and March, the previous month's subject matter and the following month's content were discussed. With reference to the former, suggestions were offered by the writer and other members of the group on ways of overcoming certain difficulties met by the teachers. During the discussions of the contents for the coming month, the teachers were instructed concerning the placing of emphasis on meaning and understanding and were cautioned each time against moving to another phase of mathematics before the students demonstrated their ability to work the problems under discussion. The materials from the textbook were outlined by the teachers and a general discussion on ways and means of teaching the material was carried out by the group.

An illustration. An example of these discussions is the presentation of fractions. Before the students were allowed to work with addition, subtraction, multiplication, or division of fractions, a thorough lesson on the parts of a fraction was discussed with the students. Such things as denominator, numerator, common fraction, improper fraction, and the meaning of a fraction were explained and discussed in detail before moving into the next phase of mathematics.

The teachers were allowed to use material they would ordinarily have used in presenting the subject matter but at no time were they allowed to use any of the so-called "modern" mathematics in their program.

Determining Pupil Achievement

Standardized achievement tests were administered in September, 1962 and in April, 1963 in order to determine pupil growth.

September, 1962 testing. In September, 1962, the Science Research Associates Test of Arithmetic Achievement for Grade Eight, Form A, was distributed to the teachers. Each teacher administered his tests to the students and returned the tests and the answer sheets to the writer. The tests were scored by the writer and each child's raw score in Arithmetic Reasoning, Arithmetic Concepts, and Arithmetic Computation, as well as his total raw score, were recorded on a card. The card for each child showed his arithmetic scores, name of teacher, school attended, and date of testing.

April, 1963 testing. At the conclusion of the experiment, Form B of the same SRA Achievement Test was administered to the experimental and control groups. Each student's raw scores on Form B for Arithmetic Reasoning, Arithmetic Concepts, and Arithmetic Computation, as well as his total raw score, were recorded on the same card used to record the September test results. Since each card contained scores for September and April, each child's progress was easily determined by comparing the two sets of scores.

The results of these comparisons are presented in the next chapter for each quartile of the experimental and control groups.

CHAPTER III

ORGANIZATION AND ANALYSIS OF THE RESULTS OF THE STUDY

The progress of the students during the interval between the September and April testing dates was computed in terms of gains or losses in raw scores for each student and terms of differences of gains in mean scores for the experimental and control groups in each quartile. Analysis of variance was applied to the data to determine the significance of the differences between the means.

Determining and Comparing the Mean Scores

As explained in Chapter II, the raw scores on Form A of the test were used to divide the 305 students in the experimental group and the 305 students in the control group into quartiles that were equated to the extent that differences in achievement between the two groups in each quartile at the beginning of the study were very small.

The gains and/or losses were next determined for each student. The data recorded on each student's index card included the raw scores made on the achievement tests administered in the fall and the spring. Scores recorded for each student were on Arithmetic Reasoning, Arithmetic Concepts, Arithmetic Computation, and the Total Raw Score for Form A and Form B. The progress of each child was determined by comparing the scores made on Form A with the scores made on Form B. The data for each child and each quartile are shown in Appendix G.

Analysis of Statistical Data

In order to test the significance of the differences in gains between the groups, the mean gain in total test scores for each teacher in each quartile for both the experimental and control group was determined. These data are shown in Tables VI, VII, VIII, and IX.

The analysis of variance was applied to determine the significance of the differences between groups. The null hypothesis that there was no significant difference between the means was rejected at the .05 level.

The analysis of variance and the resulting F (variance ratio) are shown in the paragraphs that follow for each quartile in Arithmetic Reasoning, Arithmetic Concepts, Arithmetic Computation, and total raw scores. In addition, the analysis of variance was applied to the difference between the mean gains of the total experimental and control group (rather than by quartiles).

Analysis of Data on Arithmetic Reasoning

Mean differences between the experimental and control groups on Arithmetic Reasoning are shown in Table X, page 37, for each of the four quartiles tested. The differences in mean gain for the control group was larger in each of the first three quartiles, whereas the difference in Quartile IV favored the experimental group.

The F-test for the significance of differences between means

TABLE VI

DATA SHOWING TOTAL RAW SCORE MEANS FOR THOSE STUDENTS IN QUARTILE I, FOR EACH TEACHER BASED ON RESULTS OF SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

September, 1962 A	April, 1963 B	September, 1962 A	April, 1963 B
EXPERIMENTAL GROUP		CONTROL GROUP	
43.8	54.6	33.0	53.0
45.7	55.0	49.0	75.0
43.8	53.9	45.1	69.0
48.7	56.3	44.7	55.6
43.7	53.7	44.9	59.4
38.0	41.0	43.7	56.7
42.8	50.3	45.2	50.9
44.1	53.5	43.2	54.0
40.6	51.8	40.0	77.5
44.4	62.8	41.8	67.4
43.7	63.2	38.6	47.6

TABLE VII

DATA SHOWING TOTAL RAW SCORE MEANS FOR THOSE STUDENTS IN QUARTILE II, FOR EACH TEACHER BASED ON RESULTS OF SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

September, 1962 A	April, 1963 B	September, 1962 A	April, 1963 B
EXPERIMENTAL GROUP		CONTROL GROUP	
58.0	74.6	57.0	79.0
58.5	59.8	55.7	69.7
57.6	64.0	59.7	78.5
59.6	74.9	59.0	76.3
59.8	62.8	59.0	67.6
57.2	67.8	60.0	68.0
57.3	70.3	58.4	63.0
57.2	62.2	55.5	64.5
56.2	71.2	55.5	71.8
56.9	76.8	55.7	77.5
59.0	82.0	58.2	80.7
		60.5	85.8

TABLE VIII

DATA SHOWING TOTAL RAW SCORE MEANS FOR THOSE STUDENTS IN QUARTILE III, FOR EACH TEACHER BASED ON RESULTS OF SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

September, 1962 A	April, 1963 B	September, 1962 A	April, 1963 B
EXPERIMENTAL GROUP		CONTROL GROUP	
71.4	71.8	74.1	90.9
71.6	66.5	68.3	76.5
69.6	74.5	72.6	87.0
72.6	81.0	72.4	87.3
70.3	75.9	67.4	76.6
73.9	77.1	72.0	94.0
69.5	85.0	66.0	73.0
74.7	85.8	69.5	70.0
72.0	78.8	68.8	83.8
71.8	87.4	69.7	82.7
71.0	95.0	72.2	99.1
		73.0	87.3

TABLE IX

DATA SHOWING TOTAL RAW SCORE MEANS FOR THOSE STUDENTS IN QUARTILE IV FOR EACH TEACHER BASED ON RESULTS OF SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

September, 1962 A	April, 1963 B	September, 1962 A	April, 1963 B
EXPERIMENTAL GROUP		CONTROL GROUP	
88.3	102.1	88.2	104.3
84.4	95.0	91.5	96.0
92.3	107.8	93.8	103.1
93.9	104.3	96.5	114.7
85.1	87.1	89.0	96.6
91.1	100.8	93.0	103.4
88.0	92.5	87.3	90.0
83.6	92.6	83.3	89.3
82.0	108.0	83.0	87.0
		88.3	110.0
		87.0	104.0
		91.0	106.3

was made for each of the four quartiles. None of the differences were found to be significant. These data are shown in Table XI, page 38.

TABLE X

DIFFERENCES BETWEEN MEAN GAINS IN ARITHMETIC REASONING, QUARTILES I, II, III, IV--SRA ACHIEVEMENT TEST IN ARITHMETIC

Quartile	Experimental	Control
I	3.78	4.99
II	2.69	3.84
III	2.15	3.09
IV	4.52	2.88

Analysis of Data on Arithmetic Concepts

Table XII, page 39, shows the differences between mean gains on Arithmetic Concepts for the quartiles tested. These data show that the control group made a larger gain than the experimental group in each of the four quartiles.

The difference favoring the control group in Quartile III was the only one found to be significant when the F test was applied. It was significant at the .05 level. Results of the tests for significance are shown in Table XIII, page 40.

Analysis of Data on Arithmetic Computation

The differences between mean gains on Arithmetic Computation for each quartile are shown in Table XIV, page 41. These data reveal that the differences in gains favored the control group in each of the four quartiles tested.

TABLE XI

ANALYSIS OF VARIANCE FOR ARITHMETIC REASONING--

SRA ACHIEVEMENT TEST IN ARITHMETIC

Source	Degrees of Freedom	Sum of Squares	Means Squared
Quartile I			
Between Groups	1	8.04	8.04
Between Teachers within Groups	20	194.57	9.73
	F = 8.04/9/73 = .83		
Quartile II			
Between Groups	1	7.60	7.60
Between Teachers within Groups	21	195.02	9.286
	F = 7.60/9.286 = .82		
Quartile III			
Between Groups	1	5.14	5.14
Between Teachers within Groups	21	265.08	12.62
	F = 5.14/12/62 = .41		
Quartile IV			
Between Groups	1	13.81	13.81
Between Teachers within Groups	19	249.34	13.12
	F = 13.81/13.12 = 1.052		

When the F-test was applied to these data, the difference favoring the control group in Quartile I was found to be significant at the .05 level. Results of the application of the F-test for the four quartiles on Arithmetic Computation are shown in Table XV, page 41.

TABLE XII

DIFFERENCE BETWEEN MEAN GAINS IN ARITHMETIC CONCEPTS--

SRA ACHIEVEMENT TEST IN ARITHMETIC

Quartile	Experimental	Control
I	3.74	5.40
II	3.96	4.19
III	3.00	4.80
IV	2.81	3.52

Analysis of Data on Total Scores for each Quartile

The difference between mean gains for the total score in each quartile is shown in Table XVI, page 42. The control group had the greater difference in Quartiles I, II, and III. Only in Quartile IV, which consisted of those students who ranked highest in total raw scores on the initial test, did the experimental group have a greater difference between the mean gains than did the control group.

TABLE XIII

ANALYSIS OF VARIANCE FOR ARITHMETIC CONCEPTS--

SRA ACHIEVEMENT TEST IN ARITHMETIC

Source	Degrees of Freedom	Sum of Squares	Means Squared
Quartile I			
Between Groups	1	15.22	15.22
Between Teachers within Groups	20	152.09	7.604
	F = 15.22/7.640 = 2.002		
Quartile II			
Between Groups	1	.30	.30
Between Teachers within Groups	21	139.20	6.628
	F = .30/6.628 = .045		
Quartile III			
Between Groups	1	18.60	18.60
Between Teachers within Groups	21	57.00	2.71
	F = 18.60/2.71 = 6.863*		
Quartile IV			
Between Groups	1	2.56	2.56
Between Teachers within Groups	19	37.23	1.959
	F = 2.56/1.959 = 1.306		

*Significant at the .05 level.

TABLE XIV

DIFFERENCES BETWEEN MEAN GAINS IN ARITHMETIC COMPUTATION--

SRA ACHIEVEMENT TEST IN ARITHMETIC

Quartile	Experimental	Control
I	2.91	7.28
II	4.85	7.78
III	3.03	5.36
IV	4.09	4.44

TABLE XV

ANALYSIS OF VARIANCE FOR ARITHMETIC COMPUTATION--

SRA ACHIEVEMENT TEST IN ARITHMETIC

Source	Degrees of Freedom	Sum of Squares	Means Squared
Quartile I			
Between Groups	1	105.16	105.16
Between Teachers within Groups	20	285.81	14.29
	F = 105.16/14.29 = 7.358*		
Quartile II			
Between Groups	1	49.23	49.23
Between Teachers within Groups	21	453.01	21.57
	F = 49.23/21.57 = 2.28		
Quartile III			
Between Groups	1	31.18	31.18
Between Teachers within Groups	21	545.40	25.97
	F = 31.18/25.97 = 1.20		
Quartile IV			
Between Groups	1	.64	.64
Between Teachers within Groups	19	183.64	9.665
	F = .64/9.665 = .07		

*Significant at the .05 level.

When the F-test was applied to the three differences favoring the control group and the difference (Quartile IV) favoring the experimental group, the results showed only one to be significant. The difference favoring the control group in Quartile I was at the .05 level. These data are shown in Table XVII, page 43.

TABLE XVI

DIFFERENCES BETWEEN MEAN GAINS IN TOTAL SCORE FOR ARITHMETIC REASONING, ARITHMETIC CONCEPTS, ARITHMETIC COMPUTATION, SRA ACHIEVEMENT TEST IN ARITHMETIC

Quartile	Experimental	Control
I	10.62	17.90
II	11.74	15.68
III	8.22	13.52
IV	11.28	11.07

Analysis of Data on Total Scores for the Entire Experimental and Control Groups.

The mean gains of the three hundred five students in the experimental group and the three hundred five in the control group amounted to 10.42 for the former and 14.47 for the latter.

When the F-test was applied, this difference of 4.05 was found to be significant beyond the .05 level. The results of testing are shown in Table XVIII, page 44.

TABLE XVII

ANALYSIS OF VARIANCE ON TOTAL SCORES, SRA ACHIEVEMENT TEST IN ARITHMETIC

Source	Degrees of Freedom	Sum of Squares	Means Squared
Quartile I			
Between Groups	1	291.64	291.64
Between Teachers within Groups	20	1133.06	56.653
	$F = 291.64/56.653 = 5.147$*		
Quartile II			
Between Groups	1	89.40	89.40
Between Teachers within Groups	21	1011.03	48.14
	$F = 89.40/48.14 = 1.857$		
Quartile III			
Between Groups	1	161.12	161.12
Between Teachers within Groups	21	1174.92	55.95
	$F = 161.12/55.95 = 2.879$		
Quartile IV			
Between Groups	1	.23	.23
Between Teachers within Groups	19	828.59	43.61
	$F = 23/43.61 = .005$		

*Significant at the .05 level.

TABLE XVIII

ANALYSIS OF VARIANCE ON TOTAL SCORES FOR THE EXPERIMENTAL AND CONTROL GROUPS--SRA ACHIEVEMENT TEST IN ARITHMETIC

Source	Difference	Sums of Squares	Means Squared
Between Groups	1	36316	36316
Between Teachers within Groups	87	452400	5200

$F = 36316/5200 = 6.98*$

Significant at the .05 level.

Summary

Briefly summarizing, Chapter III has presented the organization and analysis of data for the six hundred ten students used in this study. An attempt was made to show the mean achievement of the students in the experimental and control groups before the study began and mean gains at the conclusion of the study. The analysis of variance was applied to the data to determine the significance of the difference between the means for the experimental and control groups.

The data showed eighteen comparisons between the experimental and control groups. In sixteen of the comparisons the control group made the greater gains while in two areas the experimental group made greater gains. When the F-test was applied to the differences favoring each group, only four of the differences were found to be significant at the .05 level (one of these met the test beyond the .05 level). All of the significant differences favored the control group. They were: (1) Arithmetic Concepts, Quartile III; (2) Arithmetic Computation, Quartile I; (3) Total Raw Scores by Quartiles, Quartile I; and (4) Total Score for the entire experimental and control groups.

Further examination of the data revealed differences in the effects of the program on the lowest and highest ability groups. In the lowest ability group (Quartile I), the students who participated in the program planned for the control group did consistently better in the four areas tested than the students who participated in the program planned for the experimental group. In two of these areas the differences favoring the control group were significant at the .05 level. In the highest ability group (Quartile IV), the students in the experimental group did slightly better or about as well as the control group in the four areas tested.

CHAPTER IV

SUMMARY AND CONCLUSIONS

The purpose of this study was to determine the effects of a specially planned mathematics program on pupil achievement in eighth grade mathematics. This specially planned program combined the traditional course with content carefully selected from the "new" or "modern" mathematics. (See Appendixes A through F.)

Twenty-three eighth grade teachers and six hundred ten eighth grade pupils from twelve white public schools in St. Landry Parish, Louisiana, participated in the study. The twenty-three teachers were paired and grouped on the basis of degree held, years of teaching experience, years of teaching eighth grade mathematics, and special training in the "new" mathematics. There were eleven teachers in the experimental group and twelve teachers in the control group. The students were grouped into four quartiles on the basis of the SRA Achievement Test in Arithmetic, Form A, administered in September, 1962. For the experimental group, there were eighty students in Quartile I, seventy-three in Quartile II, seventy-eight in Quartile III, and seventy-four in Quartile IV. The range of scores was from twenty-six, lowest in Quartile I, to one hundred seventeen, highest in Quartile IV. For the control group, there were seventy students in Quartile I, seventy-five in Quartile II, seventy-five in Quartile III, and eighty-five in Quartile IV. The range of scores was from twenty-seven, lowest in Quartile I, to one hundred twenty-five, highest in Quartile IV.

The teachers in the experimental group met with the writer on the first Monday of each month from October through April. At these meetings the teachers were given the lesson plans they were to use for the coming month. (See Appendixes A through R.) A brief discussion of the plans was conducted and the teachers were instructed to use no more than the normal class period for teaching purposes and to supplement the textbook content with the "new" mathematics supplied by the plans. The teachers were free to use any teaching aids which they would ordinarily have used and were encouraged to allow time for the students to discuss the material rather fully before proceeding to the next phase.

The teachers in the control group met with the writer on the second Monday of each month from October, 1962 through April, 1963. These meetings consisted of discussions on the textual materials which had been taught during the previous month and those which were to be taught during the next month. The teachers were helped to place emphasis on meaning and understanding and were encouraged to allow students the opportunities to demonstrate their ability to work the problems or concepts under discussion before moving to the next phase of mathematics. The teachers were allowed to use any teaching aids they would ordinarily have used but were instructed to use only the subject matter content found in their textbooks.

In April, 1963 Form B of the SRA Achievement Test in Arithmetic was administered to all the students who participated in the study. On

The basis of the results of this test as compared with Form A which was given in September, 1962, a comparison of the mean achievement of the students in each quartile was made.

Summary of Data

The analysis of variance was used to analyze the results of this study. The means between groups and between teachers within the groups were analyzed to determine the significance of differences between the means. The null hypothesis that there was no significant difference between the means was rejected at the .05 level.

Summary of data on arithmetic reasoning. On arithmetic reasoning the control group had a greater difference in mean gain in the first three quartiles, whereas the experimental group had a mean gain difference greater than the control group in Quartile IV. None of these differences were significant at the .05 level.

Summary of data on arithmetic concepts. On arithmetic concepts, the control group had a greater difference between means in each of the four quartiles tested. The difference was significant at the .05 level only in Quartile III.

Summary of data on arithmetic computation. On arithmetic computation, the control group had larger gains than the experimental group in every quartile. In Quartile I, the difference was significant at the .05 level. The difference between means was greater in Quartile I, Arithmetic Computation, than in any other section of the test or in any other quartile.

Summary of data on total score for each quartile. When the differences were compared on the total test scores for each quartile, the control group had larger gains than the experimental group in Quartiles I, II, and III while the experimental group gained more than the control group on Quartile IV.

The tests for significance showed only one of the differences to be significant--Quartile I, favoring the control group.

Summary of data on total scores for the entire experimental and control groups. When the mean gain of the entire experimental group was compared with the mean gain of the entire control group, the data revealed that the control group did significantly better than the experimental group beyond the .05 level.

Conclusions

From the data presented in Chapter III and summarized above, the following conclusions seem warranted:

1. The achievement of the control group was higher when measured by a conventional test than the achievement of the experimental group.
2. The program used with the control group seemed better adapted to the needs of the students in the three lower quartiles.
3. The smaller differences in Quartile IV indicated that

the experimental program seemed better adapted to the higher ability group than the three lower groups.

4. Further experimentation with a larger sample, over a longer period of time, and with a variety of evaluative procedures is needed to determine the effect of the "new" mathematics content on student achievement at all ability levels.

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APPENDIX

APPENDIX A

UNIT I. FRACTIONS

I. PRIME FACTORS

I. OBJECTIVES:

A. To develop in the students the skills necessary for finding the L. C. M. by using the factoring method.
B. To develop in the students the correct technique for finding the prime factors of large numbers.
C. To provide the students with a usable definition of a prime number.
D. To teach the students to work problems for themselves by trying various methods before asking for aid.

II. INTRODUCTION:

Three volunteers to come to board to work previous night's homework. (Work only two or three problems in this fashion, having students explain the mathematical operations they used in solving the problems).

III. SUBJECT MATTER:

A. The factoring method of finding the L. C. M. involves the use of prime numbers. Read and explain the definition of a prime number. The numbers 2, 3 and 5 are examples of prime numbers. None of them have factors other than 1 and the number itself. (N.1 = N). Let's list all the prime numbers greater than 1 and less than 50.
(2,3,5,7,11,13,17,19,23,29,31,37,41,43,47)

B. A number which has factors other than one and itself. It is the opposite of a prime number. If you factor out a composite number you get its prime numbers. A composite number is formed by multiplying the prime numbers.
6 is composite. Its prime numbers are 3 and 2.

C. List all of the prime numbers in the following set of numbers. Then circle the composite numbers. (Square all prime numbers.)

53 58 59 61 63 67 69 71 73 77 78 79

D. You can express 12 as the product of its factors: 4 X 3 or 2 X 6. How would you express 12 as a product of its prime factors. That is, each of its factors should be a prime number.
(2 X 2 X 3)

E. You know 4 and 3 are factors of 12. Is 4 a prime number? (no) What are the prime numbers of 4? (2 X 2) So we can write 12 as 2 X 2 X 3. Are 2 and 3 prime numbers? (yes) (Unique factorization - to lowest prime no.)

F. You can express 20 as the product of its factors: 10 X 2. But is 10 a prime number? (no) What are the prime numbers of 10? (5 X 2) So the prime numbers of 20 are? (5 X 2 X 2)

G. Sometimes it is not easy to find the prime numbers. Therefore you should factor it in steps.
Example I.
To express 144 as the product of its prime factors we must factor in steps.

A. 144
2 X 72
2 X 6 X 12
2 X 2 X 3 X 2 X 2 X 3
2 X 2 X 2 X 2 X 3 X 3

B. 144
2 X 72
2 X 2 X 36
2 X 2 X 3 X 12
2 X 2 X 3 X 3 X 4
2 X 2 X 3 X 3 X 2 X 2
2 X 2 X 2 X 2 X 3 X 3

Example II. (if necessary)
Express 300 as a product of its prime numbers.
300
2 X 150
2 X 2 X 75
2 X 2 X 3 X 25
2 X 2 X 3 X 5 X 5
(2 X 2 X 3 X 5 X 5) = 300
This is called unique factorization because we factor a number to its lowest prime factors.

Example III. (if necessary)
Express 70 as a product of its prime factors.
70
7 X 10
7 X 5 X 2
(7 X 5 X 2) = 70

H. One method of finding prime numbers is by the Erasthostones method. List all of the odd numbers in succession. Then erase every third number after three, every 5th number after 5 and so on.

I. Homework:

Find the prime factors of the following:

1. 16 (2 X 2 X 2 X 2)
2. 18 (2 X 3 X 3)
3. 24 (2 X 2 X 2 X 3)
4. 30 (2 X 3 X 5)
5. 48 (2 X 2 X 2 X 2 X 3)
6. 64 (2 X 2 X 2 X 2 X 2 X 2)
7. 80 (2 X 2 X 2 X 2 X 5)
8. 288 (2 X 2 X 2 X 2 X 2 X 3 X 3)
9. 44 (2 X 2 X 11)
10. 90 (3 X 3 X 5 X 2)

IV. ACTIVITIES:

A. Optional Work
1. List all of the prime numbers between 2 and 100.
2. List all of the prime numbers between 125 and 150.

B. Aiding students with individual work.
Never directly answer a problem but merely suggest various other methods of working the problem. Don't let the students depend on you for an answer but only for aid.

II. FACTORING TO FIND THE L. C. M.

I. OBJECTIVES:

A. To develop in the students the ability to use factoring as a method of finding the L. C. M. for a set of numbers.

B. To provide a sufficient number of examples so that even the slow students understand.

C. To continue aiding the pupils in doing more efficient individual work.

D. To teach the students how to sign papers before turning them in.

II. INTRODUCTION:

A. Review the past few days' work orally with pupils. Explain a problem on equal fractions, one on finding the prime factors, and one on finding the L. C. M.

1. Complete this set of numbers so that each is an equal fraction.

1. $\frac{\quad}{3}$ $\frac{4}{?}$ $\frac{16}{24}$ $\frac{24}{?}$ $\frac{\quad}{123}$

2. $\frac{1}{}$ $\frac{7}{42}$ $\frac{5}{}$ $\frac{}{48}$ $\frac{}{60}$

2. Find the L. C. M. of these numbers.

   1. 9, 18, 6, 4 (36)

   2. 5, 15, 25 (75)

3. Express this number as the product of its prime factors.

   96
   2 X 48
   2 X 2 X 24
   2 X 2 X 2 X 12
   2 X 2 X 2 X 2 X 4
   2 X 2 X 2 X 3 X 2 X 2

III. SUBJECT MATTER:

A. Now that you have learned how to express a number as a product of its prime factor, you can use prime factors to find the L. C. M. of a set of numbers. Set: A set is a group, family or collection of objects. A group of numbers to be factored all belong to a set.

Example I.
First express each number as a product of its prime factors.

Number	Prime factor
12 =	2 X 2 X 3
10 =	2 X 5
9 =	3 X 3

Second, check the factors of each number to see how many times a factor is used in each number. You can see that the greatest number of 2's of any number is two. The greatest number of 5's is one and the greatest number of 3's is two.

Then the L. C. M. 12, 10, and 9 must have 2 as a prime factor twice, 5 once and 3 twice. Are 12, 10 and 9 factors of this product? (yes, 2 X 2 X 3 X 3 X 5 = 180)

B. Example II.
Find the L. C. M. of 6, 14, and 21.
First factor out each number.
6 = 3 X 2
14 = 2 X 7
21 = 7 X 3

What is the maximum number of times 2 appears? (1) 3? (1) 7? (1)
What is the L. C. M.? (2 X 3 X 7 = 42)

C. Example III. (if necessary)

Find the L. C. M. of 8, 12, and 15.
Factor each.
8 = 2 X 2 X 2
12 = 2 X 2 X 3
15 = 5 X 3
What is the maximum number of times 2 appears?
(3) 3? (1) 5? (1) 7? (0)
What is the L. C. M.?
(2 X 2 X 2 X 3 X 5 = 120)

IV. ACTIVITIES:

A. Have students factor various numbers at home to determine the L. C. M.
B. Allow time for students to discuss any problems which seem to give them the most difficulty. (It is wise to encourage discussion by the slower students.)

III. EQUAL FRACTIONS

I. OBJECTIVES:

A. To develop in the students an understanding of the basic terminology for fractions.

B. To develop in the students the skills necessary for common fractions, both largest and smallest.

C. To develop an understanding that multiplying and dividing a fraction by the same number doesn't change its value.

D. To show the relationship between improper fractions and mixed numbers.

E. To teach the students how to use diagrams to show equality of fractions.

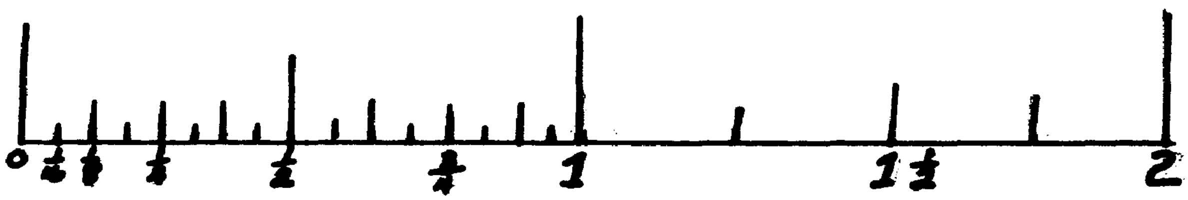
II. INTRODUCTION:

A. Vocabulary - Write on board.

1. Equal fractions--two fractions are equal if they have the same numerical value.
2. Proper fractions--two fractions are proper if they have a larger denominator than numerator.

3. Improper fractions--two fractions are improper if the numerator is equal to or larger than the denominator.
4. Mixed number--a fraction is a mixed number if it contains both a whole number and a fraction.

B. Oral:
The teacher will draw on the board a number line and ask the following questions:



1. This number line is divided into halves, fourths, eighths, and sixteenths. How many eighths are there between 0 and 3/4? (6) Are 6/8 and 3/4 equal fractions? (yes) Are 12/16 and 3/4 equal fractions? (yes)

2. How many fourths are there between 0 and 1? (4) How many eighths? (8) How many sixteenths? (16) Do 4/4 and 1 name the same number? (yes) 8/8 and 1? (Yes) 16/16 and 1? (yes)

III. SUBJECT MATTER:

1. As you know numbers such as 3/4 and 6/8 are called proper fractions. Write these on the board and then ask the class for other examples. Explain the definition of a proper fraction.

2. Fractions such as 8/8 and 24/16 are examples of improper fractions. Write these on the board and then ask the class for more examples. Explain the definition of an improper fraction.

3. Fractions such as $1\frac{1}{2}$ and 2 1/8 are examples of mixed numbers. Ask the class for more examples and write them on the board. Explain the definition of a mixed number.

4. The number three-fourths is named by the numeral "3/4". In this numeral 3 is the numerator and 4 is the denominator. These are the terms of the fraction. The numerator indicates the number of parts of the whole under discussion. The denominator names the number of equal parts into which the whole is to be divided.

What are the terms of the fraction 2/3 (2 and 3), of 15/16 (15 and 16), of 1/12? (1 and 12).

5. Look at the terms of the fraction 12/18. Is 2 a common factor of both numerator and denominator? (yes) Is 3 a common factor? (yes) Is 6? (yes) Can you think of any other common factor? The factor 6 is called the largest common factor of 12 and 18 because it is the largest number which will go evenly into both. Divide both terms of 12/18 by 6/9, 4/6, and 2/3 equal fractions? (yes)

6. Could anyone show us by means of a free hand diagram that 2/3 = 4/6 - 6/9?

7. Does 8/12 - 2/3 $(\frac{8}{12} \div 4/4 = 2/3)$

8. Does 1/3 = 2/6 (1/3 X 2/2 = 2/6)

9. Complete this sentence: A fraction may be expressed as an equivalent fraction by ______________or by______________ both its numerator and denominator, by the same number (multiplying and dividing).

10. Work these individually.

$5/8 = 10/16 = \frac{?}{24}$ (15) $= \frac{50}{?}$ (80)

$\frac{48}{64} = \frac{24}{32} = \frac{12}{?} = \frac{?}{8} = \frac{3}{?}$ (4)

$\frac{6}{5} = \frac{18}{15} = \frac{?}{10}$ (12)

$2/3 = \frac{12}{?}$ (18) $= \frac{20}{?}$ (30)

11. What is the largest common factor of the terms of $\frac{8}{12}$? (4)

Divide both terms by 4. Is there any other number except 1 which will go evenly into both numbers? (yes, -2) Four is the largest therefore the fraction 2/3 is in lowest terms. When the terms of a fraction are divided by their largest common factor, the resulting fraction is always in lowest terms.

12. If $\frac{a}{b}$ represents a whole number greater than 0 and B represents a whole number greater than a, is $\frac{a}{B}$ a proper fraction? (yes) Is $\frac{b}{a}$ an improper fraction? (yes) Does $\frac{5a}{5b}$ name the same number as $\frac{a}{b}$? (yes) Why? (because both a and b are multiplied by 5)

$\frac{a}{B} \times \frac{5}{5} = \frac{5a}{5b} \div \frac{5}{5} = \frac{a}{B}$

IV. ACTIVITY:

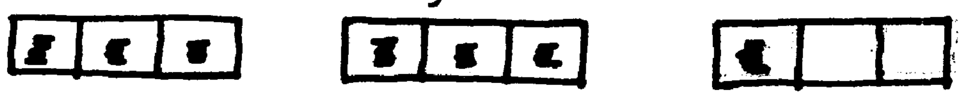
Homework

1. Copy and complete the sets of numbers so that the fractions are equal fractions.

   $\frac{1}{2}$ $\frac{?}{4}$ (2) $\frac{?}{8}$ (4) $\frac{16}{?}$ (32) $\frac{50}{?}$ (100)

   $\frac{?}{6}$ (5) $\frac{10}{12}$ $\frac{?}{24}$ (20) $\frac{50}{?}$ (60) $\frac{?}{120}$ (100)

2. The diagram shows that $\frac{7}{3}$ = (2 1/3). Supply a mixed number.



3. Which fractions are equivalent?

   1/2 2/3 4/8 1/3

   9/12 3/4 6/8 8/12

4. Write in lowest terms.

   4/8 (1/2) 3/15 (1/5) 5/20 (1/4) 7/14 (1/2)

5. OPTIONAL

   $\frac{6a}{6b} = \frac{a}{?}$ (b) $\frac{X \div 1/2}{Y \div 1/2} = \frac{?}{Y}$ (X)

IV. LEAST COMMON MULTIPLE

I. OBJECTIVES:

A. To stress the importance of finding the L. C. M. in relationship to future work.

B. To develop the skills necessary for finding the L. C. M.

C. To develop an understanding of the basic terminology of fractions.

D. To show the relationship between this material and material previously taught.

II. **INTRODUCTION:**

Vocabulary--To be written on board and explained when met in lesson.

A. Multiple--The product of two or more factors is a multiple of each of them.

B. Least common multiple--Is the smallest number which is a common multiple of all numbers being used.

Oral:

C. Finding the multiples, common multiples, and least common multiple, will be very important in finding the least common denominator and prime factors of a number. To add or subtract fractions, first we must find the least common denominator.

III. **SUBJECT MATTER:**

A. You know that 8 and 9 are factors of 72 because 8 X 9 = 72. This is to say that 72 is a multiple of 8. Refer to the definition of multiple.

Is 72 a multiple of 9? (yes, 8 X 9)
Of 6? (yes, 6 X 12)
Of 12? (yes, 12 X 6)
Of 2? (yes, 2 X 36)

Is 13 a factor of 72? (no) Then is 72 a multiple of 13? (no.)

B. You know that 20 is a multiple of both 2 and 5. Then 20 is a common multiple of 2 and 5. Is 20 the smallest common multiple of 2 and 5? (no) What is? (10)

Since 10 is the smallest number that is a multiple of 2 and 5, it is called the least common multiple (L. C. M.); check the definition of L. C. M.

C. Name four common multiples of 2 and 3. (6, 12, 18, 24)
Of 2, 3 and 4. (12, 24, 36, 48)
Which are the L. C. M.? (6 and 12)

D. Definition
The least common multiple of a set of two or more numbers is the smallest number that is a multiple of each of the numbers.
Example: 2, 3, 4 = 12.

E. Find the least common multiple. Individual work.
a. 2, 4, 5 (20)
b. 2, 4, 8 (8)

c. 3, 6, 8 (24)
d. 10, 15, 20 (60)
e. 4, 9, 18 (36)
f. 5, 10, 20, (20)

F. You have seen that the least common multiple may or may not be one of the numbers in the set, must it be the largest? (yes) If it isn't the largest, must it be a multiple of the largest number? (yes)

G. One way to find the L. C. M. of a set of numbers is first to try the largest number in the set. If it isn't a multiple of all the numbers, double it, triple it, and so on.

Example:
Find the L. C. M. of 3, 6, and 5.
1. Try 6. Is it? (no)
2. Try 2 X 6; 3 X 6; 4 X 6; 5 X 6.
3. Is 30 the L. C. M. of 3, 6 and 5? (yes)

IV. ACTIVITIES:

A. Homework: Place on blackboard. Find the L. C. M. of each of the following.
1. 8, 16, 12 (16)
2. 9, 6, 3 (18)
3. 21, 7, 6 (42)
4. 5, 15, 25 (75)
5. 4, 8, 5 (40)
6. 9, 18, 6, 4 (36)
7. 9, 12, 15 (180)
8. 4, 6, 10, 15 (60)
9. 5, 8, 10 (40)
10. 7, 5, 2, 1 (35)

APPENDIX B

UNIT II. FRACTIONS AND RATIONAL NUMBERS

I. MULTIPLYING FRACTIONS AND MIXED NUMBERS

I. **OBJECTIVES:**

A. To develop in the students the ability to multiply mixed numbers and fractions.

B. To teach the students the procedure and importance of reviewing work.

C. To teach the student how to change mixed numbers into fractions and fractions into mixed numbers.

D. To develop in the student second method of multiplying mixed numbers and fractions.

II. **INTRODUCTION:**

A. Reteach the meaning of the distributive law of mathematics.

B. Have two or three students go to the board and work some problems where they will use the distributive law of mathematics.

III. **SUBJECT MATTER:**

A. Multiply these numbers:

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{\overset{1}{\cancel{3}}}{\cancel{9}}}{\underset{8}{\cancel{16}}} \times \frac{5}{\underset{2}{\cancel{6}}} = \frac{(5)}{16}$$

As this example shows, the product of three or more fractions can be reduced to lowest terms before multiplying.

By what common factor did we divide 2 and 16? (2) 3 and 9? (3) 3 and 6? (3) Can $\frac{5}{16}$ be reduced? (No)

B. Express the products in lowest terms.

1. $\frac{3}{8} \times \frac{5}{9} \times \frac{1}{10} = \frac{1}{48}$ 2. $\frac{3}{4} \times \frac{3}{8} \times \frac{8}{9} = \frac{1}{4}$ 3. $\frac{5}{7} \times \frac{14}{15} \times \frac{3}{10} = \frac{1}{5}$

4. $\frac{6}{7} \times \frac{21}{32} \times \frac{1}{10} = \frac{3}{64}$ 5. $\frac{5}{12} \times \frac{14}{15} \times \frac{6}{7} = \frac{1}{3}$ 6. $\frac{5}{6} \times \frac{3}{10} \times \frac{2}{3} = \frac{1}{6}$

C. One way to find the product of a fraction and a mixed number or of two mixed numbers is to express the mixed numbers as improper fractions and then multiply.

Express $\frac{1}{3} \times 2\frac{5}{8}$ and $2\frac{1}{3} \times 4\frac{1}{5}$

$$\frac{1}{3} \times 2\frac{5}{8} = \frac{1}{\cancel{3}_{1}} \times \frac{\overset{7}{\cancel{21}}}{8} = \frac{7}{8}$$

$$2\frac{1}{3} \times 4\frac{1}{5}$$

$$\frac{7}{\cancel{3}} \times \frac{\overset{7}{\cancel{21}}}{5} = \frac{49}{5}$$

Express 49 as a mixed number:

$$\frac{49}{5} = 9\frac{4}{5}$$

D. Find the products: Express improper fractions as mixed numbers.

1. $\frac{1}{5} \times 3\frac{1}{8} = \frac{5}{8}$ 2. $\frac{3}{5} \times 9\frac{2}{3} = 5\frac{4}{5}$ 3. $\frac{3}{8} \times 4\frac{2}{3} = 1\frac{3}{4}$

4. $3\frac{5}{6} \times 1\frac{1}{5} = 4\frac{3}{5}$ 5. $2\frac{4}{7} \times 1\frac{5}{9} = 4$ 6. $2\frac{4}{5} \times 2\frac{1}{7} = 6$

E. Here is a second way of multiplying mixed numbers. Use it only when you think it is convenient.

$$\frac{1}{2} \times 4\frac{1}{2}$$

Is this equal to $(\frac{1}{2} \times 4) + (\frac{1}{2} \times \frac{1}{2})$? (yes)

Then multiply the $\frac{1}{2}$ in $4\frac{1}{2}$ by $\frac{1}{2}$

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Next multiply the 4 by $\frac{1}{2}$.

$$\frac{1}{2} \times 4 = 2$$

Now add both products

$$\frac{1}{4} + 2 = 2\frac{1}{4}$$

F. Using this method work the following:

1. $\begin{array}{r} 16 \\ \times 2\frac{1}{2} \\ \hline 40 \end{array}$ 2. $\begin{array}{r} 10 \\ \times 5\frac{1}{8} \\ \hline 51\frac{1}{4} \end{array}$ 3. $\begin{array}{r} 6\frac{1}{5} \\ \times \frac{5}{6} \\ \hline 5\frac{1}{6} \end{array}$ 4. $\begin{array}{r} 20 \\ \times 3\frac{4}{5} \\ \hline 76 \end{array}$ 5. $\begin{array}{r} 6\frac{6}{7} \\ \times \frac{1}{3} \\ \hline 2\frac{2}{7} \end{array}$ 6. $\begin{array}{r} 5\frac{5}{9} \\ \times \frac{3}{5} \\ \hline 3\frac{1}{3} \end{array}$

G. Work the following

1. $\frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$

2. $\frac{8}{11} - \frac{7}{10} = \frac{3}{110}$

3. $\frac{3}{4} + \frac{1}{5} = \frac{19}{20}$

4. $\frac{2}{9} + \frac{1}{4} = \frac{17}{36}$

5. $\frac{7}{8} \times \frac{3}{14} = \frac{3}{16}$

6. $\frac{3}{16} \times \frac{4}{9} = \frac{1}{12}$

IV. ACTIVITIES:

A. A general review

Put examples of all problems on the board and then answer all questions.

1. $\frac{2}{?5}$ $\frac{4}{10}$ $\frac{?10}{25}$ $\frac{16}{?40}$ $\frac{40}{?100}$

Complete so as to make all fractions equal.

2. Find the least common multiple.

9, 18, 6, 4 = 36

3. Express each of the following as a product of its primes.

1. 64 — 2 x 2x2x2x2x2
2. 72 — 3x3x2x2x2
3. 73 — 73

4. Using factoring find the Least Common Multiple.

1. 10, 12, 16 (60)

5. Work the following

a. $3\frac{5}{8}$
$+\ 3\frac{3}{20}$
$3\frac{7}{12}$
$10\frac{43}{120}$

b. $201\frac{1}{21}$
$-\ 100\frac{5}{14}$
$100\frac{29}{42}$

c. $\frac{4}{5} \times \frac{15}{32} = \frac{3}{8}$

d. $3\frac{1}{8} \times 24 = 75$

II. MULTIPLICATION AND RECIPROCALS

I. OBJECTIVES:

A. To develop in the student the ability to find the reciprocal of any number.

B. To teach the students that the multiplier or operator times its reciprocal equals to 1.

C. To reteach the associative law of multiplication.

II. INTRODUCTION:

A. Reciprocals are important.
The definition of a reciprocal is that number which gives a product of one when multiplied by the original number.
Put definition and an example of the use of reciprocal on the board.

B. We will use reciprocals in working with dividing fractions and later on in equations and inequalities we will come in contact with them again.

III. SUBJECT MATTER:

A. Find the product of these numbers.

a. 1/2 x 2 = 1

b. 3 x 1/3 = 1

c. 17 x 1/17 = 1

d. 1/286 x 286 = 1

You will find that the product of each is 1.

"Pairs of numbers whose product is 1 are called reciprocals. The reciprocal of 8 is 1/8 and the reciprocal of 1/8 is 8. Each member of the pair is the reciprocal of the other.

B. Replace each ? with the reciprocal.

1. 6 x ? (1/6) = 1	5. 1/10 x (10)? = 1
2. (1/14)? x 14 = 1	6. (1/64)? x 64 = 1
3. (1/9)? x 9 = 1	7. 298 x (1/298) ? = 1
4. 1/18 x (18)? = 1	8. 1/1764 x? (1764) = 1

C. In this example 1/4 is the multiplier.

1/4 x 32 = 8

What is 4 x 8 = 32 ?

Which number is the multiplier? (4)

D. Which is the multiplier in this example?

(1/16) x 64 = 4

What is 16 x 4 = (64) ?

Which is the multiplier? (16)

E. Which is the multiplier in this example?

(1/5) x 125 = 25

What is 5 x 25 = (125) ?

Which is the multiplier (5) ?

F. To see the relationship between these multipliers find the product of each pair of multipliers.

1/4 x 4 = 1　　1/16 x 16 = 1　　1/5 x 5 = 1

Are the multipliers in each pair reciprocals? (yes)

G. Is this sentence true? (yes)

(1/4 x 32) x 4 = 32

Then you would rewrite the problem as:

(1/4 x 4) x 32 = 32

1/4 x 4 = 1

Would any number (N) multiplied by the product of the reciprocals equal the number? (yes)

N X (1/a x a) = N

H. Using the reciprocals rewrite these problems.

(1/16 ? x 16) x 64 = 64

(1/5 x ?5) x 125 = 125

I. The commutative principle of multiplication states that the product of numbers are the same, regardless of their order. Have someone read the definition from their book. Since multiplication is associative, is this sentence true? (yes)

1/16 x (16 x 64) = 64

J. Rewrite these using reciprocals.

(Do orally)

(1/4)? x (4 x 32) = 32

1/5 x (? 5 x125)= 125

K. Find the products of each.

2/3 x 3/2 = 1   5/8 x 8/5 = 1   13/16 x 16/13 = 1

Why are these numbers reciprocals?

(Because their products are one.)

L. Rewrite as true sentences. Check by completing the indicated operations.

1. ? 3/2 x (2/3 x 15) = 15

2. 5/8 x (? 8/5 x 20) = 20

3. 1/7 x (?7 x a) = 9

4. ? 3/4 x (4/3 x 12) = 12

IV. ACTIVITIES:

A. For the pupils who need extra practice at working the problems write these problems on the board and help them work them.

1. 1/6 x (? 6 x 36) = 36

2. ? 7 x (1/7 x 21) = 21

3. 8 x (?1/8 x 24) = 24

4. 2/3 x (?3/2 x 12) = 12

III. RECIPROCALS

I. OBJECTIVES:

A. To teach the students how to divide fractions.

B. To teach the students that the multiplication of a reciprocal is equal to division by that number.

C. To develop in the students the ability to set up a written problem.

D. To teach the students the ability to identify reciprocals.

II. INTRODUCTION:

A. You have learned that the inverse of multiplication is division.

1/4 x 936 = 9

36 ÷ 1/4

$\frac{36}{1} \times 4/1 = \frac{144}{1} = 144$

III. SUBJECT MATTER:

A. Divide these problems by substituting the reciprocal for the divisor and multiply. Do these on the board.

1. 56 ÷ 14 = 4
2. 82 ÷ 1/6 = 492
3. 608 ÷ 4/5 = 760
4. 1056 ÷12 = 88
5. 14 ÷ 1/7 = 98
6. 82 ÷ 2/3 = 123
7. 1608 ÷ 24 = 67
8. 287 ÷ 7/8 = 328

B. Division is also the inverse of multiplication.

48 ÷ 6 = 1/6 x 48

63 ÷ 9 = 1/9 x 63

Don't you get the same answer by multiplying by the reciprocal as by dividing by a number?

C. To be done individually

1. 5/6 ÷ 5/8 = 1 1/3
2. 1/6 ÷ 4 1/3 = 1/26
3. 8/16 ÷ 7/8 = 9 1/3
4. 3/4 ÷ 9/10 = 5/6
5. 7/8 ÷ 15/16 = 14/15
6. 2/3 ÷ 8/15 = 1 1/4 (3/4)
7. 6 3/8 ÷ 3/4 = 8 1/2
8. 2 5/32 ÷ 2 5/8 = 23/28

D. The band leader said that one-eighth of the students in school were in the band. There were 97 students in the band. How many students were there in the school?

You know that the band was one-eighth of the student body. So 1/8 of the student body = 97.

The entire student body should be 8/8. So multiply 8 x 9 to find the total number of students.

8 x 97 = 776.

What is the relationship between 1/8 and 8? (Reciprocals.) When you knew 1/8 of the students equaled 97, you multiplied 97 by the reciprocal of 1/8 to find the total student body.

E. If 234 is 1/10 of a number, what is the number?

1/10 x ? = 234

Why can (10 x 234) replace the ??

10 x 1/10 is 1 & 234 = 234

Complete the operation in parentheses.

$$\begin{array}{r} 234 \\ \underline{\times\ 10} \\ 2340 \end{array}$$

Was 2340 the original number? (yes)

Try it.

$1/\cancel{10}_{1} \times \cancel{2340}^{234} = \frac{234}{1}$

F. Use this method to rewrite the following as true sentences.

1. 7/8 x ? = 1806 (2064)

2. 11/12 x ? = 264 (288)

3. 5/6 x ? = 1435 (1722)

4. 2/7 x ? = 1764 (6174)

5. 3/5 x ? = 2841 (4735)

6. 2/3 x ? = 3922 (5883)

G. One year a dealer was able to sell only 9/10 of the new cars shipped to him. If he sold 234 cars in all, how many cars were shipped to him? (260)

9/10 x ? = 234

$9/10 \times \frac{10}{\cancel{9}_{1}} \times \cancel{234}^{26}) = 234$

9/10 x 260 = 234

H. Mr. Greene's family spent 7/8 of the money he budgeted for clothes. If there was $112 left, what was the original amount set aside? ($896)

7/8 x ? = 112

$7/8 \times (\frac{8}{\cancel{7}_{1}} \times \cancel{112}^{16}) = 112$

7/8 x 1

1/8 x ? = 112

1/8 x (8/1 x 112) = 112

1/8 x 896 = 112

I. Tell whether or not the two numbers are reciprocals of each other.

1. 5/2 , 2/5

2. 1/12, 12

3. 19/2, 9 1/2

4. 4/17, 71/4

5. 5 3/4, 5 4/3

6. 6 2/3, 21/32

7. 23/12, 21/32
8. 7/3, 21/9
9. 5/5, 5/5
10. 12/10, 5/6
11. 0/3, 0/4
12. 9/10, 1 1/9

IV. ACTIVITIES:

A. for the pupils who want extra practice at working problems using reciprocals to find the whole, write these problems on the board and help them work them.

1. 5/9 x ? = 895 (1611)
2. 3/4 x ? = 186 (248)
3. 11/10 x ? = 1122 (1020)
4. 10/12 x ? = 5000 (6000)
5. 7/1 x ? = 1442 (206)
6. 2/5 x ? = 8422 (21055)

IV. DIVIDING FRACTIONS

I. OBJECTIVES:

A. To teach the students a different method of division.

B. To give practice at working with written problems.

C. To show the relationship between decimal fractions and common fractions.

D. To provide work reviewing the previous day's lessons.

II. INTRODUCTION:

A. You have learned that to divide common fractions you multiply by the reciprocal.

1 3/4 ÷ 5/8

7/4 ÷ 5/8

$7/\cancel{4}_{1} \times \overset{2}{\cancel{8}}/5 = 14/5$ or 2 4/5

B. Here is a second method of working this problem. In this method the fractions are expressed with a common denominator, then one numerator is divided by the other.

1 3/4 ÷ 5/8 =

7/4 ÷ 5/8 =

14/8 ÷ 5/8 =

14 ÷ 5 = 2 4/5

Here is a second example.

3/5 ÷ 3/10

6/10 ÷ 3/10

6 ÷ 3 = 2

C. Could we have three people come to the board and work some examples?

1. 15/16 ÷ 3/4 = 1 1/4

2. 2 3/8 ÷ 1 1/16 = 2 4/17

3. 1 2/3 ÷ 5/9 = 3

III. SUBJECT MATTER:

A. Paper from a book is about 1/320 of an inch thick. About how many sheets of this paper would there be in a 3/4-inch book? (Ask pupils to work this at their desks and afterwards get someone to explain it.)

3/4 ÷ 1/320

$\frac{240}{320}$ ÷ 1/320

240 ÷ 1 = 240 pages

B. You know that fractions may be expressed either as common fractions or as decimal fractions. For example the common fraction 1/10 may be expressed as .1.

Express as decimal fractions.

3/10	.3
1/100	.01
4/1000	.004

C. Mixed numbers, as you know, may be expressed as decimal mixed numbers. For example 1 1/10 may be expressed as 1.1. Express 2 7/10 as a decimal mixed number. (2.7)

D. Here are the place names for the numbers right and left of the decimal point.

Hundred thousands	Ten-thousands	Thousands	Hundreds	Tens	UNITS	Tenths	Hundredths	Thousands	Ten-thousands	Hundred thousands
8	8	8	8	8	8	8	8	8	8	8

E. There are four decimal places in .8675. How many zeros are there in the denominator of the equivalent common fraction? (Four)

$.8675 = \frac{8675}{10,000}$

F. Compare the number of decimal places in H. with the number of zeros in the denominators of the equivalent common fraction. What is the pattern?

(The number of zeros always equals the number of decimal places.)

G. Express the following as common fractions: (orally)

1) .32612 $\frac{32612}{100,000}$

2) .07 $\frac{7}{100}$

3) .1006 $\frac{1006}{10,000}$

4) .006 $\frac{6}{1,000}$

5) .6 $\frac{6}{10}$

H. Express these common fractions as decimal fractions. (Orally)

1. $\frac{651}{10,000}$ (.0651)

2. $\frac{1242}{100,000}$ (.01242)

3. $\frac{4}{10}$ (.4)

4. $\frac{2}{10,000}$ (.0002)

I. Express the following decimal mixed numbers as mixed fractions. (Orally)

1. 26.42 $26\frac{42}{100}$
2. 19.011 $19\frac{11}{1000}$
3. 48.503 $48\frac{503}{1000}$
4. 70.00064 $70\frac{64}{100,000}$

IV. ACTIVITIES

A. Homework

These problems are a brief review of some of the problems dealing with fractions. This is due in two days.

1. Perform the indicated operations.

A. 40 3/10 + 20 2/15 + 5 7/12   B. 756 13/16 + 349 5/6 + 902 7/12   C. 482 1/3 - 194 5/7   D. 806 4/9 - 328 4/5

2. Find the products:

A. $\frac{21}{25}$ x 5/7   B. $\frac{16}{39}$ x 3/4   C. 3/8 x 5/16 x 16/45

D. 5 1/9 x 3/10

3. Rewrite, replacing each "?" with a reciprocal.

A. ? X (14 X 83) = 83   B. 1/12 X (? X 403) = 403

4. Using Reciprocals, make these true sentences.

A. 1/4 x (?) = 37   B. 1/7 X (?) = 112   C. 5/9 X (?) = 60

5. Give the quotients.

A. 1/6 ÷ 7/8   B. 5/12 ÷ 5/6   C. 7 1/3 ÷ 11/15

D. 27 1/2 ÷ 6 2/3   E. 9 1/5 ÷ 3 5/6   F. 2 7/32 ÷ 4 7/8

Paper from a book is about 1/320 of an inch thick. About how many sheets of paper would be in a 3/4 inch book?

V. POLYNOMIALS

I. OBJECTIVES:

A. To show the students how a decimal fraction may be expressed as a whole number times a fraction.

B. To teach the use of exponents in expressing decimal fractions.

C. To help students learn that skipping steps in mathematics only leads to confusion.

D. To provide extra problems for those wanting more practice.

II. INTRODUCTION:

A. There is a very close relationship between fractions and decimal fractions. We will learn to convert one into the other and then into a polynomial expression.

III. SUBJECT MATTER:

A. The number .4 can be expressed as 4 X 1/10. The number .01 can be expressed as 1/100 or 1 X 1/10 X 1/10. The number .007 can be expressed as 7 X 1/10 X 1/10 X 1/10.

B. Does 1/10 X 1/10 = $1/10^2$? (yes) Then may .01 be expressed as 1 X $1/10^2$? (yes) Does 1/10 X 1/10 X 1/10 = $1/10^3$? (yes) Then may .007 be expressed as 7 X $1/10^3$? (yes)

C. Using exponents in the denominator of the fraction, express each of the following.

1. .0008 (8 X $1/10^4$)
2. .06 (6 X $1/10^2$)
3. .00005 (5 X $1/10^5$)
4. .002 (2 X $1/10^3$)
5. .8 (8 X 1/10)

D. Study these problems. In each case, compare the number of places in the decimal fraction with the exponent in the denominator. Is there a relationship?

(yes, the exponent is always the same number as the number of decimal places.)

E. Express each of the following as a decimal fraction.

1. 6 X $1/10^3$ (.006)
4. 7 X $1/10^2$ (.07)

2. $3 \times 1/10$ (.3) 5. $4 \times 1/10^4$ (.0004)

3. $9 \times 1/10^5$ (.00009)

F. Using exponents, let us analyze the value represented by each digit in this decimal fraction .60849

The 6 is in tenth's place	$6 \times 1/10$
The 0 is in hundredth's place	$0 \times 1/10^2$
The 8 is in thousand's place	$8 \times 1/10^3$
The 4 is in ten-thousand place	$4 \times 1/10^4$
The 9 is in hundred-thousands place	$9 \times 1/10^5$

Do you see that .60849 may be expressed as a polynomial?
$(6 \times 1/10) + (0 \times 1/10^2) + (8 \times 1/10^3) + (4 \times 1/10^4) + (9 \times 1/10^5)$

G. Express each of the following as a polynomial.

1. .385 $(3 \times 1/10) + (8 \times 1/10^2) + (5 \times 1/10^3)$

2. .6752 $(6 \times 1/10) + (7 \times 1/10^2) + (5 \times 1/10^3) + (2 \times 1/10^4)$

3. .37298 $(3 \times 1/10 + (7 \times 1/10^2) + (2 \times 1/10^3) + (9 \times 1/10^4) +$ $(8 \times 1/10^5)$

4. .5304 $(5 \times 1/10) + (3 \times 1/10^2) + (0 \times 1/10^3) + (4 \times 1/10^4)$

H. Is this sentence true? (yes) Check by working out the operations.

$$(6 \times 1/10) + (7 \times 1/10^2) + (5 \times 1/10^3) + (2 \times 1/10^4) = 6752 \times 1/10^4$$

.6 + .07 + .005 + .0002 = .6752

.6
.07
.005
.0002
.6752 = .6752

I. Do you see a relationship between the exponent in the denominator of $1/10^4$ and the number of places in the decimal fraction? (.6752) (Yes, they are the same.)

J. Is this a true sentence? (Yes)

$.86024 = 86024 \times 1/10^5$. Check by multiplication. Have one student work it on the board and the rest of the class at their desks.

$86024 \times 1/10^5 = 86024 \times 1/100.000 = \frac{86024}{100,000}$

$86024 = 86024$

K. Express each of the following as a whole number times a fraction whose denominator is ten or a power of ten.

1. .315 $315 \times 1/10^3$
2. .00971 $971 \times 1/10^5$
3. .5006 $5006 \times 1/10^4$
4. .72 $72 \times 1/10^2$
5. .038 $38 \times 1/10^3$

IV. ACTIVITIES:

A. Practice exercise:

1. Change these decimal fractions into whole numbers times a power of 1/10.

a) .06 $6 \times 1/10^2$

b) .3 $3 \times 1/10$

c) .0007 $7 \times 1/10^4$

d) .005 $5 \times 1/10^3$

2. Express these as a decimal fraction.

a) $7 \times 1/10^2$ .07

b) $9 \times 1/10^5$ .00009

c) $5 \times 1/10^3$ .005

d) $3 \times 1/10^1$ .3

3. Express each as a polynomial expression.

a) .71 $(7 \times 1/10) + (1 \times 1/10^2)$

b) .301 $(3 \times 1/10) + (0 \times 1/10^2) + (1 \times 1/10^3)$

c) .60003 $(6 \times 1/10) + (0 \times 1/10^2) + (0 \times 1/10^3) + (0 \times 1/10^4) + (3 \times 1/10^5)$

d) .00573 $(0 \times 1/10) + (0 \times 1/10^2) + (5 \times 1/10^3) + (7 \times 1/10^4) + (3 \times 1/10^5)$

VI. ADDITION OF RATIONAL NUMBERS

I. OBJECTIVES:

A. To introduce the use of negative rational numbers.

B. To show a few practical uses of the number line.

positive plus positive numbers and negative plus positive numbers.

D. To state the rules in adding these three types of numbers.

E. To teach the use of the number line in doing these procedures.

II. **INTRODUCTION:**

A. You have covered the positive numbers on the number line. These are all the numbers right of the zero or origin. Today we will go into negative numbers.

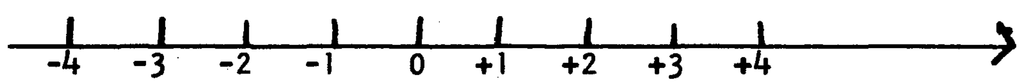
III. **SUBJECT MATTER:**

A. On a number line we have zero, the positive numbers right of the zero and the negative numbers left of the zero. A good example of this is a thermometer. We have zero degrees and positive degrees up to 110. Below zero we have negative degrees down to 20. Any temperature above zero is recorded with a a" +". and a "-" represents degrees below zero.

B. The idea of points along a line on opposite sides of a fixed point occurs frequently in our ordinary tasks. We use locations north and south of a given point such as Lafayette, We also use to the left or to the right, altitudes above or below sea level, longitudes east or west, or the time before or after a certain event.

In each of these there are points located on opposite sides of a given point.

C. To construct a number line we use zero as the center and then measure off equal distance right and left or up and down.



We locate -1 as opposite to +1 in the sense that it is 1 unit to the left of zero. Similarly -2 is opposite to +2, $-\frac{1}{4}$ is opposite $+\frac{1}{4}$. These pairs of numbers are equal distance from zero but on opposite sides.

The negative symbol "-" tells us that the number is less than zero.

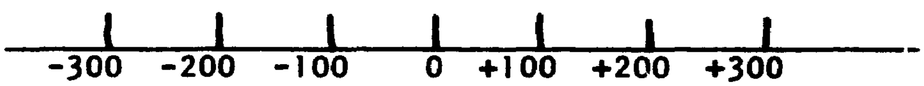
If no sign appears the number is positive. We use the positive sign '+" to emphasize positive.

D. These new numbers we have introduced by this process are the <u>negative rational numbers</u>.

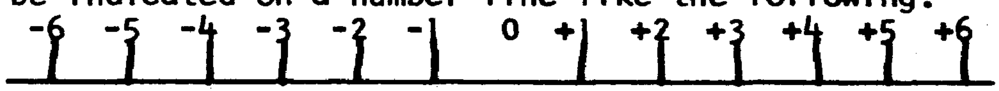
E. The set consisting of positive rational numbers, negative rational numbers, and zero is called <u>rational numbers</u>.

Uses of Negative Numbers

F. To plot an airliner flying an east-west course over Chicago we could let positive numbers represent distance east of Chicago and negative numbers represent distance west of Chicago. For an airliner flying a north-south course over Chicago, how could you represent this?



G. The time before and after the launching of a satellite can be indicated on a number line like the following.



(Seconds before launching) (Seconds after)

H. Note that the number line need not be horizontally. If we wish to show altitude above and below sea level, it is more natural to place the line how? (Vertical)

I. Locate on the number line the following numbers.

a. -8 d. 4/8

b. -7/4 e. 1.5

c. 1 3/4 f. $-\frac{1}{2}$

Are there any opposites in this list? (Yes)

-7/4 & 1 3/4, 4/8 & $-\frac{1}{2}$

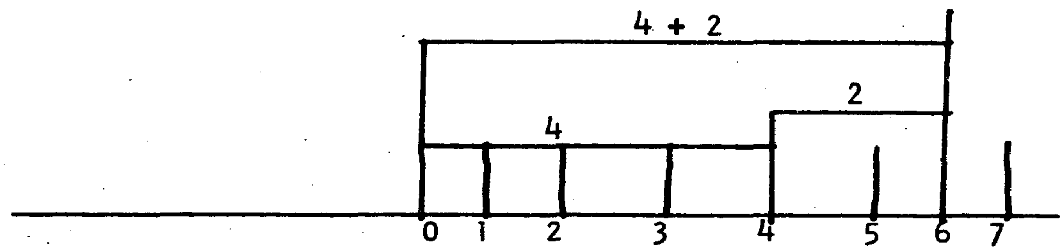
J. Arrange the following numbers in the order in which they appear on the number line.

-4, 1/4, -7/4, -7/8, 5/8, -6, -3/8, 3/4.

Addition of Rational Numbers

K. Let us say we wanted to add the two positive numbers 4 and 2.

First we draw an arrow 4 units long, then another 2 units long. When adding positive numbers, we always move to the right in the positive direction.

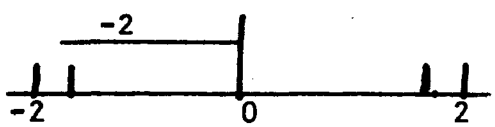


L. Find the following positive numbers.

a. 2 + 3 c. 1 + 7

b. 3 + 2

M. To construct a -2 on the number line we move 2 units to the left. -2 is the opposite of +2 on the number line because they are the same distance from 0.

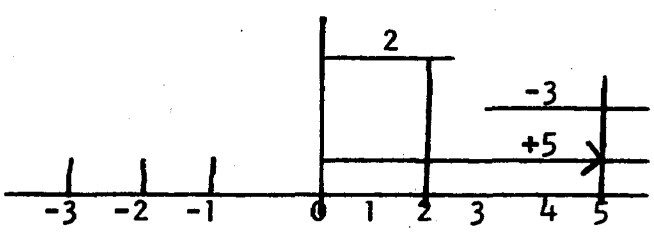


N. Find the following negative numbers.

a. -4 c. -3

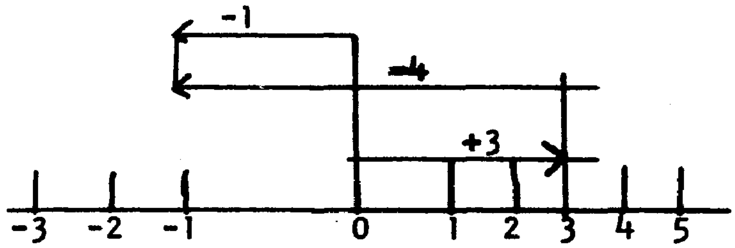
b. $-\frac{1}{2}$ d. -5/3

O. How would you sketch the sum of +5 + (-3) using directed arrows?



Thus $\pm$ 5 + (-3) = +2

P. How would you indicate +3 + (-4) ?



Thus +3 + (-4) = -1

Q. Find the sums of the following.

a. (+3) + (-2) +1

b. -1 + (-3) -4

c. -4 + (+2) -2

d. -2 + (+2) 0

R. Thus we can see that in adding positive and negative we can state a few rules.

1. When both numbers are positive the sum is positive.

+5 + (+3) = +8

2. When both numbers are negative the sum is negative.

(-5) + (-3) = -8.

3. When one number is positive and the other negative it is the number farther from the origin which determines the sign. The larger number is always farther away from 0.

(-5) + (+3) = -2

S. Find the sums and sketch, using arrows on the number line. (Get one student to come to the board and the rest of the class work at their desks).

1. 9 + (-5)

2. -8 + 11

3. +3 + (-7)

T. Supply the missing numbers. (Do orally)

1. $3 + (-3) = 0$
2. $(-4) + (-4) = 0$
3. $(-75) + 74 = -1$
4. $-(\frac{1}{2}) + (+\frac{1}{2}) = 0$
5. $1/3 + (2/3) = 1$
6. $14 + (-2) = +12$

IV. ACTIVITIES:

A. Extra problems for those desiring extra practice.

1. Supply the missing numbers.

a. +6 (+ ( ) = 0
b. (-0.45) + 0.45 =
c. 25 + (-6) =
d. (-5) + (-7) =
e. 17 + (-23) =
f. 10 + ( ) = -1
g. -4 + ( ) = -2
h. (-8) + ( ) = -16
i. (-4) + ( ) = -10

VII. COORDINATES

I. OBJECTIVES:

A. To teach the pupils to locate the point when the coordinates of the point are given.

B. To teach the students to give the coordinates if the point is given.

C. To help the students that coordinates determine distance and direction of the point.

D. To teach the labeling of the X and Y axis.

E. To teach the I, II, III and IV quadrents.

II. INTRODUCTION:

A. Review briefly that every rational number has a point on the number line.

B. Ask what three types of numbers compose the rational numbers.

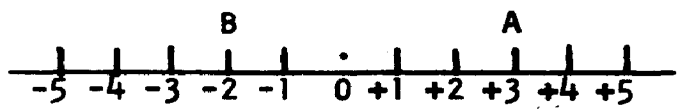
C. Ask one student to read the definition of a coordinate.

D. Review sketching of a problem to help find the sum.

III. SUBJECT MATTER:

A. We have seen that a rational number can always be associated with a point on the number line. The number associated in this way with a point is called a coordinate of the point.

B. On this number line point A is by the number (+3). Point B is by (-2). When we write A (+3) we mean that A is the point with coordinate +3. The B (-2) means that the point B has a coordinate -2.



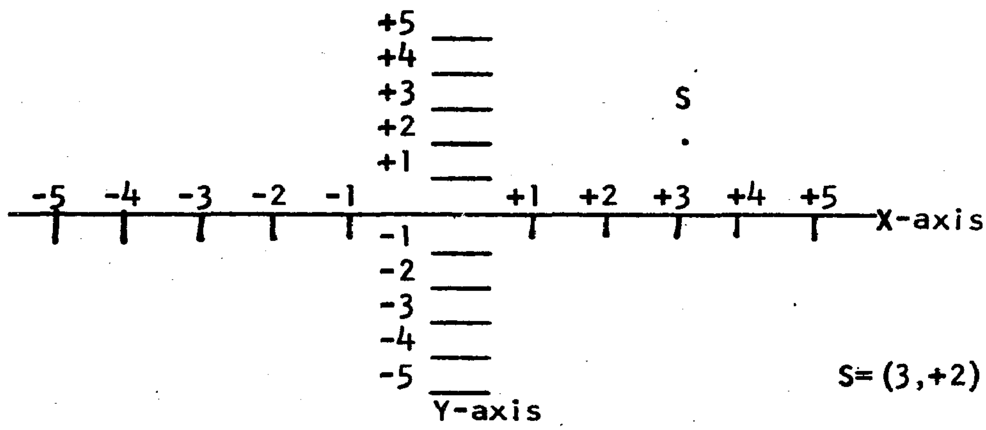
C. Now we recall that every positive number has a point on the positive side of the number line. Every negative number corresponds to a point on the negative half line. Therefore when we say the coordinate of point A is (+3) we are saying two things about A.

We are saying the <u>direction</u> from the point origin (+=right) and the distance (3).

D. Find the following points on the number line:

1. a = -1
2. b = 1
3. c = 0
4. D = 5/2
5. E = 3/2
6. How far is it from a to b?

E. You recall I said that number lines may be vertical as well as horizontal. We have also learned that each point has a coordinate. Suppose we wanted to find point S which does not lie on the number line and cannot be located by a single coordinate. So draw a vertical number line intersecting the horizontal number line at 0. We will call the horizontal number line the <u>X-axis</u> and the vertical number line the <u>Y-axis</u>. To determine the coordinate or point S, draw a line from S perpendicular to the x-axis, it intersects the x-Axis at (+3). Now draw a perpendicular line from S to the Y-axis. It intersects at (+2). Point S has a X-coordinate of (+3) and a Y-coordinate of +2. We write this as (+3, +2). We use parentheses and always write the X-coordinate before the Y-coordinate.



F. Find the following points:

1. A = (+4, +3)   3. C = (-6, -1)

2. B = (-1, +2)   4. D = (+2, -2)

This is called plotting the point.

G. Please note that it is very important that we write the X-axis first and then the Y-axis. If we do it backward we get a different coordinate.

H. Draw a double number line on your paper and locate the following points: (Have one person come plot it on the board).

1. (4,1) = A   6. (0,-5) = F

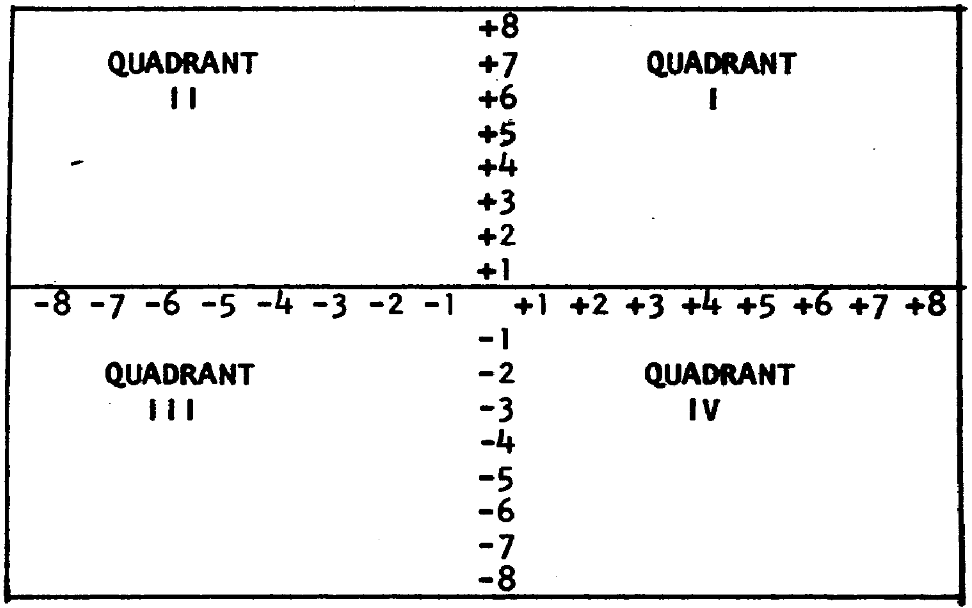
2. (1,0) = B   7. (-6,0) = G

3. (-1,-1) = C   8. (4,4) = H

4. (-3,3) = D   9. (-5,3) = I

5. (4,-3) = E   10, (6,0) = J

I. You notice that this double number line is divided into four sections. These sections are called <u>quadrants</u>. They are numbered in counter-clockwise direction. Quadrant I. includes the plane above the X-axis and to the right of the Y-axis.



J. Write the number of the quadrant in which you find the point represented by these coordinates.

Coordinate	Quadrant
1. (3,5)	?I
2. (1,-4)	?IV
3. (-4,4)	?II
4. (-3,-1)	?III
5. (8,6)	?I
6. (7,-1)	?IV
7. (-3,-5)	?III

K. Let's answer the following questions. (Orally)

1. Both numbers of the coordinates are positive = Quadrant ?I.
2. Both numbers of the coordinates are negative = Quadrant ? III.
3. The X-coordinate is negative and the Y-coordinate is positive = Quadrant ? II.
4. The X-coordinate is positive and the Y-coordinate is negative = Quadrant ?IV.

5. If the X-coordinate is Zero and the Y-coordinate is not zero, where does the point lie? (on Y-axis).

6. If the X-coordinate is not zero and the Y-coordinate is zero, where does the point lie? (on X-axis).

7. If both coordinates are zero, where is the point located? (on 0).

IV. ACTIVITIES:

A. What are the 3 rules of adding signed numbers?

B. Add these:

a. 9+ (-5) = 4

b. 10 + (-7) = 3

c. (8) + 11 = 3

d. (-12) + 7 = -5

C. Supply the missing numbers:

a. -3 + (-3) = -6

b. (+4) + (-4) = 0

c. (+6) + (-6) = 0

d. (-75) + (+74) = -1

e. 1/3 + (+2/3) = 1

APPENDIX C

UNIT III. EQUATIONS

I. WRITING NUMBER PHRASES

I. OBJECTIVES:

A. To learn some of the basic terminology of equations.

B. To be able to determine an open phrase from a closed phrase.

C. To be able to convert a number phrase into words.

D. To be able to convert a sentence into a number phrase.

II. INTRODUCTION:

A. Today we go into equations. Can someone define the word equation for us?

B. We will use what we have learned about the number line and about signed numbers. We will also use fractions to help solve our equations.

C. Today we will go into learning to read equations, determine whether they are closed or opened number phrases and being able to translate words into numbers phrases and phrases into words.

III. SUBJECT MATTER:

A. Sometimes the unknown number is not so easy to find. For example, suppose you were given this problem:

Tom bought a ticket for a football game. Altogether he paid $1.10, including the tax. If the cost of the ticket is $1.00 more than the amount of the tax, what is the amount of the tax on the ticket?

Most of you would guess 10¢. If the tax is 10¢ and the ticket $1.00, both combined are $1.10. This is not correct because if the cost of the ticket is $1.00, then that would only be 90¢ more than the tax.

Use all of the clues given to write a <u>number sentence</u>.

Let X represent the cost of the tax. If the ticket cost \$1.00 more than the tax, add (X + 1.00) to find the cost of the ticket. Now, if we add the tax (X) plus the ticket (X+1.00) we get the number sentence X + (X+1.00) = 1.10

Can you find the amount of the tax? The correct answer is \$.05 and the ticket costs \$1.05. Does \$.05 plus \$1.05 equal \$1.10?

To work the problem leave all the unknowns on one side and all the knowns on the other:
X + X + 100 = 110
X + X = 110 - 100
2X = 10
X = 5
Tax = \$.05

First we will learn how to write a number sentence and later on in the chapter we will learn how to solve number sentences.

B. A number sentence can be written like this:
X + 7 + 9

If seven is added to a certain number X, the sum is 9. This sentence is divided into two phrases: "9"; and the other phrase is "X+7". These are number phrases because each represents or describes a certain number.

If these numbers phrases describe a specific number such as: (3+5), 9, or (5-2) they are called closed number phrases.

What specific number is represented by X-4? We don't know the value of X so it could have different values. This is called an open number phrase because it is open to so many values. Examples are: (X-4), 7Y, (2+Z) and B+4.

You must be able to use the clues in the sentence or problem to change an open phrase into a closed phrase, thus solving the unknown.

C. Earlier we used this number sentence: (X+7=9). Is the value of 9 known? Is it an open or closed phrase? What about (X+7)?

D. You must be able to translate these phrases into words. The number phrase X+7 may be translated into "the number X increased by seven".
What about 7X? Is it equal to "seven times the number X"?

IV. ACTIVITIES:

A. Answer these problems orally.

1. Translate each of the following into number phrases:

   a. The sum of X and 5.
   b. The number X decreased by 3.
   c. The product of 8 and X.
   d. One-fourth of the number X.
   e. The number X increased by 10.
   f. The number X multiplied by 7.
   g. The number which is 11 subtracted from X.
   h. The number X divided by 2.
   i. The number which is 6 less than X.
   j. The number X decreased by 9.

2. If X is equal to 12 work every problem in section 1.

3. Translate these number phrases into words:

   a. X+1
   b. X-3
   c. 2X
   d. 18/X
   e. 4X

II. WRITING NUMBER SENTENCES

I. OBJECTIVES:

A. To show the relationship between everyday sentences and number sentences.

B. To teach the students how to convert number sentences into symbols.

C. To teach the students how to fill in the unknown of an equation so as to make it true.

D. To teach the students the symbols of equality and unequality.

II. INTRODUCTION:

A. This section is to give you practice instruction and practice in writing number sentences. These include equations and inequations.

B. The complete mastering of equations will be left to the ninth grade.

C. Equations are used in many different fields. They are used to design airplanes, space ships and even to predict weather.

D. Formulas are equations. What is the area of a square which is 6 feet long. ($A = 6^2$ or $A = L.W.$)

III. SUBJECT MATTER:

A. All of us use sentences in talking and when we read, we read sentences. In mathematics we deal with many kinds of sentences. Consider these sentences: The sum of 8 and 7 is equal to 15. $8 + 7 = 15$
Five is greater than the sum of one and two. $5 > 1 + 2$
The sum of 3 and 4 is not equal to the product of 3 and 4.
$3 + 4 \neq 3 \times 4$

B. We see that each sentence contains two numbers phrases connected by a verb or a verb phrase. In number sentences, verbs are represented by the symbols "$=$", "$<$", and "$>$", and "$\neq$".

C. In a number sentence the symbol for equality is "$=$". In In this sentence $X+3 =8$; $X+3$ and 8 represent the same number. When X is 5 the sentence is true; if X is not 5 then the sentence is false.

D. In a number sentence "$X-4 > 7$", "$>$" means greater than. This means that the number representing "$X-4$" is greater than 7. This sentence shows inequality. Other examples of inequality are "$<$", smaller than and "$\neq$", not equal.

E. Some sentences are true and others are false. "Do orally."
The sun sets in the West
$4+5 \neq 3 \times 3$
$3+2 > 4$
Abe Lincoln was the first president.

F. Some sentences can be true or false depending upon the value placed. Consider this one: "George was the first president."
$X+3=8$
If it is not necessarily true or false it is an <u>open sentence</u>.
Washington and 5 are the solutions to these open sentences.

G. Can you determine the solution for this inequality $X-4>7$? How large must X be in order for this inequality to be true? This is true only if X is any number greater than 11.

H. Do the following orally:

1. $X + 3 = 5$	2
2. $Y + 3 > 5$	$Y > 2$
3. $K + 13 = -15$	-28
4. $M + 25 = 21$	6
5. $S + 25 < 31$	$S < 6$
6. $T + 10 \neq 5$	$T \neq -5$
7. $X + (-7) = 2$	9
8. $Y + (-7) > 2$	$Y > 9$
9. $N + (-9) = -2$	7
10. $X + (-3) = 6$	9
11. $P + (-15) = -1$	14
12. $X + (-15) < -1$	$X < 14$
13. $4b = 12$	$b = 3$
14. $4a \neq 12$	$a \neq 3$
15. $5w = 35$	$w = 7$
16. $5n < 35$	$n < 7$
17. $13X = -13$	$X = -1$
18. $7Y = -56$	$Y = -8$

IV. ACTIVITIES:

A. Homework

1. Translate each of the following number sentences into symbols and find the value of the unknown.

a. The number X increased by 5, is equal to 13.
$X + 5 = 13$ $\quad X = 8$

b. The number 3 subtracted from X is equal to 7.
$X - 3 = 7$ $\quad X = 10$

c. The product of 8 and X is equal to 24.
$X \cdot 8 = 24$ $\quad X = 3$

d. When X is divided by 4 the quotient is 9.
$X/4 = 9$ $\quad X = 36$

e. Ten more than the number X is 31.
$10 + X = 21$ $\quad X = 11$

f. The number X multiplied by 7 is equal to 25.
$X \cdot 7 = 35$ $\quad X = 5$

g. The number 11 subtracted X equals minus 5.
$X - 11 = -5$ $\quad X = 6$

h. The number 6, less than X is 15.
$X - 6 = 15$ $\quad X = 21$

i. The number X divided by 2 is equal to 7.
$X/2 = 7$ $\quad X = 14$

III. GRAPHS OF SOLUTIONS

I. OBJECTIVES:

A. To teach the students the ability to sketch solutions.

B. To increase the student's ability to solve inequalities.

C. To give the students practice at adding, subtracting, dividing and multiplying signed numbers.

D. To teach solution sets, not just solutions.

II. INTRODUCTION:

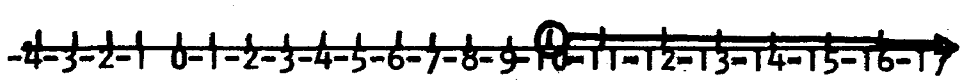
. We have learned how to determine open and closed phrases and open and closed sentences. Today we will learn to sketch the solutions so as to better understand the solution to equations.

III. SUBJECT MATTER:

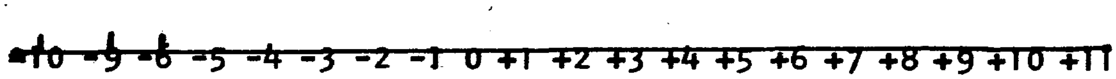
A. We can sketch the solutions to our equations on a number line by association the numbers with points on a line. Let's sketch the points (0, +3, +6)
It's sometimes helpful to sketch the solutions to equations.

B. In this open sentence, X+3=8 what is X? (5) On our number line we put a dot on the solution, 5.

C. In the sentence we have an inequality X-4 >7. What is the set of solutions? (>11). We would sketch this solution like this:



D. What is the solution set of this inequality? X+ 4 > (5). How would you sketch this one?



E. What is the solution set of X+3=3+X? We find that any number we choose will replace X. So the solution is the set of all numbers. This is the way we sketch this solution.

F. How will we sketch the following solution sets? (Do orally with one volunteer at the board)

1. +2
2. -3
3. 4>
4. <-3>
5. 0, +1, +3, +2, +4, +5>
6. +1, +2, +3, +4, +5>
7. <-1
8. <-1>
9. _2> 0<
10. $^{-}3$, -2, -1, 0, +1, +2, +3

G. We are still not ready to work very complicated equations or inequalities. We will go into more complicated ones at the end of this chapter and when you study algebra next year.

H. Do these orally: (Find the solution for each)

1. X+2 = 6	4
2. 4 + X = 0	-4
3. 2X = 6	3
4. 2X x 10	4
5. 4 - X>1	-3

I. Sketch these answers on a number line.

IV. ACTIVITIES:

. For those requesting additional practice. (Find set and sketch)

1. X+1=1+X	<0>
2. Y+$^{-}$1> 0	<1
3. 1– b> 0	1<
4. 13– X=14	-1
5. 2/X = $^{-}$1	-2

IV. WRITING EQUATIONS

I. OBJECTIVES:

A. To review briefly writing solution sets on a number line.

B. To make sure the pupils understand that what is done to one side of an equation must be done to the other.

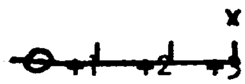
C. To teach the importance of writing carefully the number sentence or equation.

D. To teach the pupils that a step by step procedure for the problems is best method.

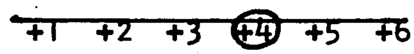
II. INTRODUCTION:

A. Let us review graphing solutions sets on a number line as this review will aid us in today's lesson.

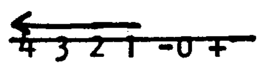
1. 3



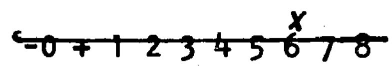
2. >4



3. <-2

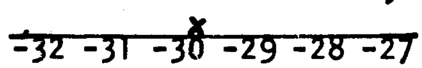


4. 2 X>12 X = 6



5. >5<

6. D/5=-6 D=-30



B. Solve this equation

13-X=14 X=-1

III. SUBJECT MATTER:

A. In solving problems, translating the English language, or words, into the number sentence is often the most important part of the task. It's often the most difficult part also. By being very careful in writing the equation we find the set of solutions is easy to find.

B. Consider this problem: "The sum of a certain number and eight is equal to two more than the product of X and the number. What is the number?

1. Let X represent the unknown number
2. Next write the two number phrases using X to represent the unknown.
3. X+ 8--first phrase
4. 4X+ 8 -- second phrase
5. X+ 8=4X+ 2--the open sentence
6. Find the number: 2

Remember that in this section we are not concerned primarily with the solution but in being able to write the open sentence or equations.

C. A train travels at 80 miles per hour. How long does it take for this train to make a 560 mile trip?

1. Let T represent the number of hours the train travels.
2. Write a phrase representing the number of miles the train travels in T hours. 80T
3. Write an equation stating the condition of the problem.

$80T = 560$

$\frac{8\cancel{0}T}{80} = \frac{56\cancel{0}}{80}$

$T = 7$

D. Mary is 14 years old. She is five years older than her brother. How old is her brother?

1. Let B represent the brother's age.
2. Write a phrase representing the problem.

$14 = B+5$

$14-5 = B+5-5$

$9 = B$

E. A boy is four years younger than his sister. If the boy is 10 years old, how old is the sister?

1. Let S represent the sister's age.
2. Write the phrase

$S-4=10$
$S-4+4=10+4$
$S= 14$

F. Work the following in class (individually) and discuss.

1. A boy bought a number of model planes costing 25¢ each. He spent 75¢. How many kits did he buy?

$K \times 25¢ = 75¢$
$K \times \frac{\cancel{25¢}}{\cancel{25¢}} = \frac{\cancel{75¢}}{\cancel{25¢}}$
$K = 3$

2. A boy's age seven years from now will be 20. How old is the boy now?

   $A - 7 = 20$
   $A + 7 - 7 = 20 - 7$
   $A = 13$

3. How many feet are there in a board having a length of 72 inches?

   $F X\ 12 = 72$
   $F \times \frac{\cancel{12}}{\cancel{12}} = \frac{\cancel{72}}{\cancel{12}}$
   $F = 6$

4. How many feet are there in a board 5 yards long?

   $F = 5 \times 3$

5. Ann was 3 years old ten years ago. How old is Ann now?

   $A - 10 = 3$
   $A - 10 + 10 = 3 + 10$
   $A = 13$

6. If three dollars is added to twice the money Dick has, the result is less than $23. How much money does Dick have?

   $2M + 3 < 23$

IV. ACTIVITIES:

1. Homework: Write the equations; mimeographed --

   a. How many dollars may be obtained in exchange for a total of 450 pennies?

   $D = \frac{450}{100}$

   b. At a certain speed a plane will travel more than 500 miles in two hours. For what speed is this true?

   $2S = 500$

   c. If one is added to twice a girl's age the result is 19. What is the girl's age?

   $2A + 1 = 19$

d. A man drove a total distance of 240 miles at an average speed of 40 miles per hour. How long did it take for the drive?

$40S = 240$

e. If a baby sitter earns 65¢ per hour, how much will she earn in 5 hours.

$65 \times 5 = M$

V. SOLVING EQUATIONS

I. OBJECTIVES:

A. To learn the definition of an equation.

B. To teach the importance of keeping an equation "balanced."

C. To teach the students that the answer must be one positive unknown.

D. To teach the students the use of the inverse operation to solve an equation.

II. INTRODUCTION:

A. Yesterday we learned to write equations. We were mostly interested in the equation and not the answer.

B. Today we will learn the procedure of solving these equations.

C. First we will solve our five homework equations.

1. $D = \frac{450}{100}$

$$100D = \frac{450 \times 100}{100}$$

$$\frac{10\cancel{0}D}{\cancel{1}00} = \frac{4\cancel{5}0}{\cancel{1}00}$$

$D = \$4.50$

2. $2S > 500$

$$\frac{2S}{2} > \frac{500}{2} \qquad S > 250$$

3. $2A+1 = 19$

$2A+1-1 = 19-1$

$\frac{\cancel{2}A}{\cancel{2}} - \frac{\cancel{18}}{\cancel{2}}$

$A = 9$

4. $\frac{40S}{40} = \frac{240}{40}$

$S = 60$

5. $65 \times 5 = M$

$322 = M$

III. SUBJECT MATTER:

A. It is of the greatest importance that we keep the equation balanced at all times. What is done to one side must be done to the other side.
SOLVE: $X + 5 = 6$

Use the inverse of $\cancel{X} + 5$. (-5)
Add to both sides $(X + 5 - 5 = 6 - 5)$
Now cancel $(X + \cancel{5} - \cancel{5} = 6 - 5)$
$X = 1$

CHECK:
$1 + 5 = 6$
$6 = 6$ (BALANCED)

B. SOLVE: $X + 6 = 5$

The inverse of $+6$ is a -6. Add to both sides
$X \cancel{+6} \cancel{-6} = 5 - 6$
Cancel $X = -1$
CHECK:
$X + 6 = 5$
$-1 + 6 = 5$
$5 = 5$

C. Get pupils to come to the board and work the following and explain.

1. $X + -7 = 7$
$X + (-7) + (-17) = 7 + (+7)$
$X = 14$

CHECK:
$X + (-7) = 7$
$14 + (-7) = 7$
$7 = 7$

2. $X + -7 = -7$
$X + (-7) + (+7) = -7 + (+7)$
$X = 0$

CHECK:
$X + -7 = -7$
$0 + -7 = -7$
$-7 = -7$

3. $4 = X + 3$
$4 - 3 = X + 3 - 3$
$1 = X$

CHECK:
$4 = X + 3$
$4 = 1 + 3$
$4 = 4$

4. $T + 6 = -13$
$T + 6 - 6 = -13 - 6$
$T = -19$

CHECK:
$T + 6 = -13$
$-19 + 6 = -13$
$-13 = -13$

IV. ACTIVITIES:

A. Homework

1. $6 \times 2 + X = 1 \times 12$	$X = 508$
2. $X + 10 = 22$	$X = 12$
3. $5X = 15$	$X = 3$
4. $\frac{1}{2}X = 17$	$X = 34$
5. $Y + 6 = 5 + 3$	$X = 2$

VI. SOLVING EQUATIONS

I. OBJECTIVES:

A. To teach the students to use the inverse of multiplication and division in solving equations.

B. To teach the use of fractions and decimal fractions in equations.

C. To teach the importance of step by step problem solving.

D. To teach the students how to change a negative unknown to a positive unknown.

II. INTRODUCTION:

A. Today we will go into more complex equations. We will also learn to make a negative unknown into a positive unknown.

B. First, we will work last night's homework.

1. $6.2 + X = 1.12$
   $6.2 - 6.2 + X = 1.12 = 6.2$
   $X = 5.08$

2. $X + 10 = 22$
   $X + 10 - 10 = 22 - 10$
   $X = 12$

3. $5X = 15$
   $\frac{5X}{5} = \frac{15}{5}$
   $X = 3$

4. $\frac{1}{2}X = 17$
   $\frac{\frac{1}{2}X}{\frac{1}{2}} = \frac{17}{\frac{1}{2}}$
   $X = 34$

5. $Y + 6 = 5 + 3$
   $Y + 6 = 8$
   $Y + 6 - 6 = 8 - 6$
   $Y = 2$

III. SUBJECT MATTER:

A. $2X + 5 = 10$
   Add -5 to both sides
   $2X + 5 - 5 = 10 - 5$
   $2X = 5$
   Divide both by 2
   $X = 2\frac{1}{2}$
   Check:
   $2X(2\frac{1}{2}) + 5 = 10$
   $10 = 10$

B. $7/C = 1$

$\frac{CX7}{C} = 1XC$

$7 = C$

Check

$\frac{C}{7} = 1$

$1 = 1$

C. Get the pupils to work the following on the board:

1. $5X + 2 = -2$
   $5X + 2 - 2 = -2 - 2$
   $\frac{5X}{5} = \frac{-4}{5}$
   $X = -\frac{4}{5}$

2. $-10X - 1 = 9$
   $-10X - 1 + 1 = 9 + 1$
   $-10X = 10$
   $-X = 1$
   $X = -1$
   Multiply both sides by -1 to get a positive unknown.

3. $2X + -3 = 9$
   $2X + -3 + +3 = 9 + 3$
   $2X = 12$
   $X = 6$

4. $2U + 1 = 11$
   $2U + 1 - 1 = 11 - 1$
   $2U = 10$
   $U = 5$

5. $3X + 10 = 5$
   $3X + 10 - 10 = 5 - 10$
   $3X = -5$
   $X = -1\ 2/3$

IV. ACTIVITIES:

. Homework

1. $19 = 6 - Y$	$Y = 13$
2. $45\ Xb = 1$	$b = 1/45$
3. $6 = X/18$	$X = 108$
4. $7 = 3X + 1$	$X = 2$
5. $14 = X = 5 \times 28$	$X = 5 \times 14$

APPENDIX D

UNIT IV. INEQUALITIES

I. ADDITION PROPERTY OF INEQUALITIES

I. OBJECTIVES:

A. To show the similarity between equations and inequalities.

B. To teach the grafting of inequalities on a number line.

C. To teach the pupils that if a number is added to both sides of an inequality we have an equivalent inequality.

D. To provide a difficult problem for the more advanced pupil.

II. INTRODUCTION:

A. What three signs have we learned that represent inequalities?

$(>, <, \neq)$

B. On a previous test we used inequalities but we used trial and error to solve them. We can use the same rules we used to solve equations to inequalities. What we do to one side we must do to the other.

III. SUBJECT MATTER:

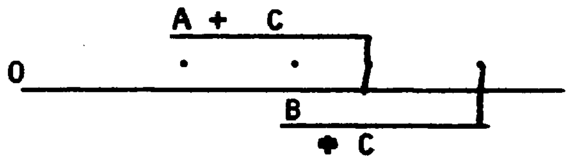
A. Take the inequality $a < b$.
Add c to both sides.
$(a + c < b + c)$

"If we add the same number to both sides of an inequality we have an equivalent inequality."

B. Now let us try numbers.
$4 < 5$
Add 2 to both sides.
$4 + 2 < 5 + 2$
$6 < 7$

C. Let's graph $a \angle b$ and a number line.

Now add C to both sides



D. Now we will work an inequality.

$X + (-3) \angle 8$

$X + (-3) + (+3) \angle 8 + 3$
$X \angle 11$

$X \angle 11$ is an equivalent inequality.
to $X + (-3) \angle 8$.

E. To be worked by the teacher and explained.

$X + 5 \angle 7$

$X + 5 - 5 \angle 7 - 5$
$X \angle 2$

F. Worked by the teacher.

$7 \angle X + 5$
$7 - 5 \angle X + 5 - 5$
$2 \angle X$

G. To be worked by an advanced pupil on the board.

$X + (-2) \angle 8$
$X + (-2) + (+2) \angle 8 + 2$
$X \angle 10$

Check it:
$X + (-2) \angle 8$
$9 + (-2) \angle 8$
$7 \angle 8$

H. Have pupils come to the board and work the following.

1. $Y + (-3) \angle 10$
$Y + (-3) + (+3) \angle 10 + 3$
$Y \angle 13$

2. $10 < Y + (-3)$
   $10 + 3 < Y + (-3) + (+3)$
   $13 < Y$

3. $2X + 4 < 5 + X$
   $2X - X + 4 < 5 + X - X$
   $X + 4 < 5$
   $X + 4 - 4 < 5 - 4$
   $X < 1$

4. $3X + 2 < 2X + (-3)$
   $3X - 2X + 2 < 2X + (-2X) + (-3)$
   $X + 2 < -3$
   $X + 2 - 2 < -3 - 2$
   $X < -5$

5. $1/2\,X + 3 > (-1/2X) + 4$
   $1/2\,X + 1/2\,X + 3 > (-1/2\,X) + (+1/2\,X) + 4$
   $1X + 3 > +4$
   $X + 3 - 3 > 4 - 3$
   $X > 1$

6. $7 + 2X < X + (-7)$
   $7 + 2X + (-X) < X + (-X) + (-7)$
   $7 + X < -7$
   $7 - 7 + X < -7 + (-7)$
   $X < -14$

IV. ACTIVITIES:

A. Mimeographed Homework

1. $X + 3 < 9$	$X < 6$
2. $X + 5 < 10$	$X < 5$
3. $10 > X + 5$	$5 > X$
4. $X + (-4) < 8$	$X < 12$
5. $12 < Y + 6$	$6 < Y$

Optional:
$1/4\,X + 4 > (-3/4\,X) + 8$ $\quad$ $4 > X$

II. SOLVING EQUATIONS

I. OBJECTIVES:

A. To teach the students how to solve equations with fractions and decimal fractions for solutions.

B. To teach the solution sets of equations which have an unknown for a denominator.

C. To reteach the rules for solving equations.

II. INTRODUCTION:

A. Today we will go into a little more complicated equations. One of these equations has an unknown for a denominator.

B. First let us correct last night's homework.

1. $19 = 6 - Y$
   $19 - 6 = 6 + 6 - Y$
   $13 = - Y$
   $13 \cdot -1 = -Y \cdot -1$
   $-13 = Y$

2. $45 \cdot B = 1$
   $\frac{45 \cdot B}{45} = 1/45$
   $B = 1/45$

3. $6 = \frac{X}{18}$
   $6 \cdot 18 = \frac{X \cdot 18}{18}$
   $6 \cdot 18 = X$
   $108 = X$

4. $7 = 3X + 1$
   $7 - 1 = 3X + 1 - 1$
   $\frac{6}{3} = \frac{3X}{3}$
   $2 = X$

5. $.14 + X = 5.28$
   $.14 - .14 + X = 5.28 - .14$
   $X = 5.14$

III. SUBJECT MATTER:

1. This problem has an unknown for a denominator. How would you solve it?

$\frac{14}{B} = 7.$

Multiply both sides by B. The opposite of dividing by B is multiplying by B.

$\frac{14 \cdot B}{B} = 7 \cdot B$

Cancel:

$14 = 7B$

Divide both sides by 7:

$\frac{14}{7} = \frac{7B}{7}$

$2 = B$

2. $\frac{8}{R} = 16$
   $8 . R = 16 . R$
   $8 = 16R$
   $\frac{8}{16} = \frac{16R}{16}$
   $\frac{8}{16} = R$
   $1/2 = R$

3. Get volunteers to work the following.

1. $\frac{56}{M} = 8$

$\frac{56.M}{M} = 8.M$

$\frac{56}{8} = \frac{8.M}{8}$

$7 = M$

2. $\frac{32}{Z} = 96$

$\frac{32.Z}{Z} = 96.Z$

$\frac{32}{96} = \frac{96Z}{96}$

$1/3 = Z$

3. $1/2\ t - 1.7 = -1.3$

$1/2\ t - 1.7 + 1.7 = -1.3 + 1.7$

$1/2\ t = .4$

$2.1/2\ t = .4\ .\ 2$

$t = .8$

4. $-5\ X - 7 = 2\ X$

$-5\ X - 2X - 7 = 2\ X - 2X$

$-7\ X - 7 = 0$

$-7X - 7 + 7 = 0 - 7$

$\frac{-7X}{7} = \frac{-7}{7}$

$-1.-X = -1.-1$

$X = 1$

III. DIVISIBILITY--CASTING OUT THE NINES

I. OBJECTIVES:

A. To teach the students to work problems alone or, to become an independent worker.

B. To teach the pupils a short cut method of dividing by nine.

C. To develop an understanding of why this method works.

D. To make the pupils draw a conclusion from these problems.

II. INTRODUCTION:

A. Can you tell me which of these numbers is divisible evenly by 9.?

1. 333	Yes
2. 252	Yes
3. 3582	Yes
4. 24534	Yes
5. 558	Yes

B. Does anyone in the class have a number they want me to work to see if it is evenly divisible by 9?

III. SUBJECT MATTER:

A. You may know a simple and interesting way to tell whether a number is divisible by 9.
The rule is, a number is divisible by 9 if the sum of its digits is divisible by 9.
If the sum of its digits is divisible by 9 then the entire number is divisible by 9.

B. Let us try this number. 156,782.
First, add the digits.
1 + 5 + 6 + 7 + 8 + 2 = 29
Is this number (29) divisible by 9? **(NO)**.
Therefore the number 156,782 is not divisible evenly by 9.

C. Let's try another number. 136,782.
1 + 3 + 6 + 7 + 8 + 2 = 27
27 is divisible by 9 so the original number 136,782 is divisible evenly by 9.

E. Actually we could even have been lazier.
After we add 1 + 5 + 6 + 7 + 8 + 2 we get 29.
Now add these two numbers 2 + 9 = 11
Is 11 divisible by 9? **(NO)**.
So we know this number is not divisible by 9.

F. We could have carried it still further by adding the 1 + 1 in 11.
1 + 1 = 2
Is 2 divisible by 9? **(NO)**.

G. Let's try another set of numbers.
345,214. Add these numbers.
3 + 4 + 5 + 2 + 1 + 4 = 19
Now add 1 + 9 = 10
Now add 1 + 0 = 1
Is 1 divisible by 9.
So is 345,214 divisible evenly by 9? **(NO)**.

H. Now let's try one problem at our desk.
Use the number 245,898 and see if it is evenly divisible by 9.
2 + 4 + 5 + 8 + 9 + 8 = 36
3 + 6 = 9
Is 9 divisible by 9? (YES)
Therefore we know what?
That 245,898 is divisible by 9.

I. Why is this so?
By dividing the number 9 we could have proven whether it can be divided evenly.
But, this would give us no idea if it would work whether it would work for a different number.

To show what we are doing let us write the number in decimal notation. 156,782

1. $1X10^5 = 5X10^4 + 6X10^3 + 7X10^2 + 8x10^1 + 2 X 1 =$

   This is equal to

2. $1 X (99999 + 1) + 5 X (9999 + 1) + 6 X (999+1) + 7 X (99+1) + 8 X (9+1) + 2 x (1) =$

3. Now by the distributive property

   $5 X 9999+1 = 5 X 9999 + 5 X1$ and similarly for each number.

4. Also if addition is commutative we may add these numbers at the end.

   $1 X 99999 + 5 X 9999 + 6 X 999 + 7 X 99 + 8 X 9 + (1+5+6+7+8+2).$

5. Now we know that all these 9's are divisible by 9 and the products of all these are divisible by 9, and the sum of the products is divisible by 9.

   Hence, the original number is divisible if: 1+5+6+7+8+2 is divisible by 9. This is the sum of the original number.

   This shows that no matter what the given number is, the same principle holds.

IV. ACTIVITIES:

Homework

Method:

1. Add all the digits together.
2. If the sum is larger than 9 add the digits of the sum.
3. If this number is divisible by 9 the original number was also.
4. Check it by dividing the long or regular way.

(1) 256,428
(2) 333
(3) 42,888
(4) 51,425
(5) 246,810

When the number is not divisible evenly by 9, compare the remainder when the sum of the digits is divided by 9 with the remainder when the original number is divided by 9. Can you guess a general fact? What?

IV. EQUATIONS--GRAPHING SOLUTIONS OF TWO UNKNOWNS

I. OBJECTIVES:

A. Finding ordered pairs of solutions.

B. Teaching the pupils to graph solutions of equations.

C. Teaching the pupils to graph inequalities.

D. Teaching the pupils to graph equations on curved lines.

E. Teaching the pupils to graph inequalities on curved lines.

F. Reteaching coordinates on number lines.

II. INTRODUCTION:

A. So far we have dealt with equations and inequalities with one unknown.

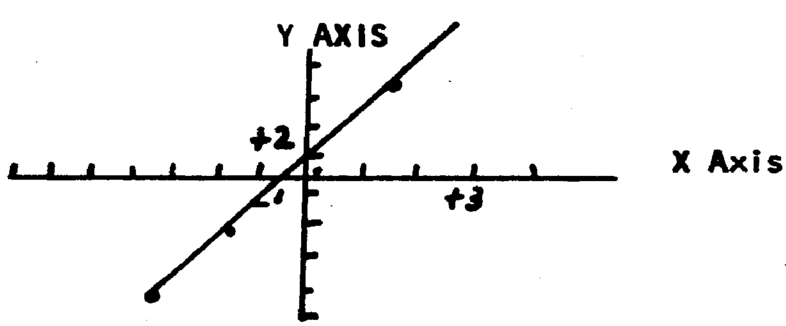
Let's try this equation.
$X + 1 = Y$
If $X = 3$ and $Y = 5$ this equation is correct but there are various other solutions. Fill in a chart of a few possibilities.

X	Y
-7	-6
0	1
1	2
2	3

Thus we have various possible solutions. How can we incorporate all possibilities?

III. SUBJECT MATTER:

A. Let us use a double number line. Let X be the horizontal line and Y be the vertical one. If we use our various solutions as coordinates and connect these with a straight line we have what?



This straight line incorporates all the possible solutions for the equation $X + 1 = Y$.

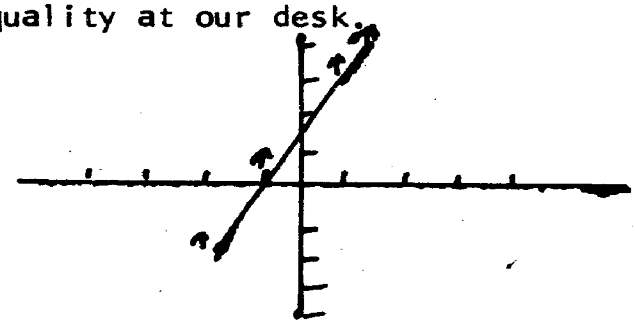
B. Let's try another equation.

$2X + Y = -1$
Let $X = -3$ (This gives the equation)
$2(-3) + Y = -1$
$-6 + Y = -1$
Solving the equation:
$6 + (-6 + Y) = 6 + -1$ (by the addition property)
$(6 + -6) + Y = 5$ (by the association property)
$Y = 5$

C. Now let us try an inequality at our desk.

$Y > X + 1$

X	Y
0	>1
1	>2
2	>3
-1	>0



Be sure to designate the greater and less than signs. All possible solutions for the inequality Y X + 1 are above this line.

D. Graph this at your desk.

$Y < X - 2$

E. Let us try a square root. Example: $Y = X^2$

X	Y
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

F. Let's try an inequality.

$$Y > X^2$$

X	Y
-4	16
-3	9
-2	4
-1	1
-0	0

IV. ACTIVITIES:

Homework: Work and Graph

1. $Y = X + 1$
2. $Y = X - 1$
3. $Y = X^3$
4. $Y > -1 \, . \, X^2$

V. EQUATIONS AND INEQUALITIES--SENTENCES, STATEMENT SET

I. OBJECTIVES:

A. To prepare the students for a strong math background.

B. To help the students learn the new terminology.

C. To distinguish between a statement and a sentence.

D. To help the students realize what a set is.

II. INTRODUCTION:

A. Review the basic concepts with the students. This will help you to know how much they already understand.

III. SUBJECT MATTER:

Mathematical Sentence

1. Expresses a complete thought.
2. Examples (Mimeographed)
   Tell whether each of the following is a sentence.

A. $4-8 = 4$
B. $\triangle ABC$
C. $6 + 8$
D. 5 Pencils
E. $-3 + (-5) = -8$
F. She has a dress
G. $6 + 8 > 10$
H. $2 + 5 < 6$

3. Sentences that are true or false

A. $8 + (-3) = 5$ (T)
B. $8 + 6 < 15$ (T)
C. $-7 (-3) > -21$ (T)
D. $-9 \div (-3) = -3$ (F)
E. All airplanes have 2 engines (F)
F. It has 4 legs (cannot tell)
G. She is 5 feet tall (cannot tell)
H. He is a U. S. senator (cannot tell)

B. STATEMENT:

1. A sentence that is either definitely true or definitely false
2. Examples (Mimeographed)
Tell which of the following sentences are statements

A. The largest state is Rhode Island.
B. He invented the electric light.
C. $6 \times 8 < 6 + 8$
D. $6 \times 8 > 6 \pm 8$
E. $\square + 8 = 6 \mp 8$
F. $\triangle \times 1 = -12$
G. This number is an even number
H. $\square$ was a great mathematician

C. Open Sentences and variables

1. A sentence that contains an unknown is an open sentence.
2. A variable is the unknown itself. Example $\triangleleft \times 12 = -1$ is an open sentence; $\triangleleft$ is the variable.
3. Examples (Mimeographed)
What is the variable in each of the following open sentences
A. <u>He</u> wore a green hat
B. <u>It</u> contains hydrogen
C. $\square + 9 = 12$
D. $\triangle + 4 = 7$
The temperature is $\square$ degrees
E. <u>She</u> was the wife of a judge
F. $\underline{Y} \times 4 = 20$
G. $9 \geq \underline{N} + 5$

D. SET:

1. A set is any collection of items
Example:
   a. Set of all the boys in the class
   b. Set of all the natural numbers
   c. Let the children give examples of sets.
2. A set is signified between brackets { } and read thusly
   Set A is a set whose numbers are --------
3. Finite Set- A set that has a definite end. 1,2,3,4.
4. Infinite Set-- A set with no end. {1,2,3,4,.......}
5. Replacement Set.
   a. The set in which you would expect to find the answer to a problem.
   b. Example: For an open sentence "$N < 6$", an appropriate replacement set in the set of all numbers.
6. Solution set
   a. The solution to the problem or the answer.
   b. Example: For the open sentence $X + 2 \geq 8$ the solution set is {6, 8, 10,...}
   c. Exercise:
   For each of the following open sentences, use for the replacement set {3,4,5,6,7,8,9,10} . In each case, find a replacement for the variable that will result in a true statement.

(1) $\triangleleft + 2 < 6$	3
(2) $? \times 2 = 10$	5
(3) $\square \times 2 > 19$	10
(4) $X + 6 < 9\frac{1}{2}$	3
(5) $24 = 6\ Y$	4
(6) $Z + 6 = 14 - Z$	4
(7) $(2 + \lozenge) + 1 = 9$	4
(8) $Y + 3\frac{1}{2} > 9$	9
(9) $15 - Z > 10$	3,4

Homework in Activities

IV. ACTIVITIES:

A. Class discussion, let the pupils give examples, call a different one to be sure they all understand.

B. Homework:
For each of the following open sentences, use the set {0, 1, 2, 3} as the replacement set. Find all the replacements, if there are any, that make each open sentence a true statement.

A. $5 + X = 6$	1
B. $5 + W > 6\frac{1}{2}$	2,3
C. $5 - Y = 4$	1
D. $\frac{5 \cdot X4}{20} \times Z = 3$	3
E. $2q - q = 3$	3
F. $2 + \triangle = 2$	0
G. $X \div 1 = X$	0,1,2,3
H. $\frac{x}{8} = 2/16$	1
I. $5 = Z > 10$	none
J. $8 \times \triangle = 8$	1
K. $r - r = 0$	0,1,2,3
L. $\frac{Y}{Y} = 2$	None

D. Give a 5 or 10 minute quiz.

. Using the set of positive whole numbers as the replacement set, tell which of the following open sentences are true for all numbers of the replacement set.

(True) A. $\square + 5$ is a positive whole number.
(False) B. $13 - \square$ is a positive whole number.
(True) C. $Z \times 3 = 3 \times Z$
(False) D. $Y \div 4 = 4 \div Y$
(True) E. $X + 12 = 12 + X$
(False) F. $Z - 3 = 3 - Z$

VI. MATHEMATICS--EQUATIONS AND INEQUALITIES

I. OBJECTIVES:

A. To familiarize the pupils with the correct mathematical vocabulary.

B. To help the children learn coefficients.

C. To show the difference between equivalent equations, and equations that are not equivalent.

D. To lay a foundation for higher mathematical work.

E. To show the children the difference between equalities and inequalities.

F. To teach the idea of "root" in order to work higher equations.

G. To instill in the pupils the meaning of member, expression and coefficient.

H. To show the students the steps necessary in discovering a basic principle.

I. To help the students think mathematically.

J. To provide the students with activities to practice these theories.

K. To show the students how these principles will help them to solve difficult equations.

L. To develop an appreciation for algebra and higher math.

M. To encourage class discussion.

N. To help the students simplify an equation.

O. To put into practice everything we have learned so far.

II. INTRODUCTION:

Review briefly the various mathematical terms learned during the previous week. Encourage participation by all students, especially the low average and slow students.

III. SUBJECT MATTER:

A. Member

1. In the case of an equation, such as $4t\ (7 - T) = 16 - t$, it is convenient to have a name for the expression at the left of the equal sign and for the expression at the right of the equal sign. Both expressions are called members of the equation. The expression $4t\ (7 - t)$ is the left member and the right member is $16 - t$.

2. Exercise:
What is the right member in each of the following equations and inequalities?

a. $3u - 5 = 19 - u$
b. $0 = W - (2 - 3w)$
c. $3x < y + 6$
d. $- 3 = y + (- 3)$
e. $2 < x + 17$
f. $18 > 2t - 4$

B. Expression

1. Any of the following

   a. A number 3, 5, 21, -8
   b. A variable W, x N
   c. The sum of two expressions $5 + x$
   d. The difference of two expressions $6 - (y+2)$
   e. The product of two expressions $3x \times (y-2)$
   f. The quotient of two expressions x/z

2. Exercise:
   Which of the following are expressions? Which are sentences?

   a. 14
   b. $x \div (4 - 2x)$
   c. $7 + 3K < -18$
   d. $x + (Y - 16)$
   e. $4 = W$
   f. $4 - 7$
   g. $5 > 3$
   h. t
   i. $(4t - u) + (5 \quad 2u)$

C. Coefficient

1. In a product such as 5y, the "5" is called the coefficient of "y". Similarly, the "y" is called the coefficient of 5. You can see that a coefficient is one of the factors of the product.

2. Exercise:
   Write the coefficient of t in each of the following expressions.

   a. $3t$
   b. $t\ (7 - t)$
   c. $(3 + 3w)t$

D. Equations and inequalities

1. When two things are equal it is called an <u>equality</u>.
   Example $3 + 4 = 7$
2. When two things are not equal it is called an <u>inequality</u>.
   Example $3 < 5$; $5 > 3$; $5 + 4 \neq 6$
3. Exercise
   Tell which of the following open sentences are equations and which are inequalities

a. $(3 \times \square) + 3 = 6$
b. $x - 4 \quad 8$
c. $\frac{1}{2} - 6 = x + 2$
d. $5 + 3$ is the same as -2
e. $5y + 2$ doesn't equal 3

4. Mimeographed homework listed in activities.

E. Roots

1. The members of the solution set is called the <u>root</u> of the equation. Example: 3 is the root of $2x = 6$.

2. Exercise:
State whether 5 is a root of each of the following equations. Make the necessary replacements to be certain that 5 does or does not satisfy each of the equations.

a. $2x - 1 = 9$
b. $3x - 1 = 13$
c. $w = 5$
d. $3x + 1 = 16$
e. $6x - 2 = 4x + 8$
f. $6 = y$
g. $4x - 8 = 12$
h. $2y + 1 = 12$
i. $5 = x$

3. Homework in activities.

4. Exercise on roots and replacement sets in activities.

F. Equivalent equations

1. If two equations are such that the solution set of one is the same as the solution set of the other, the equations are called equivalent.

2. Exercise:
In each of the following exercises, use the set of all whole numbers as the replacement set and select two equations that are equivalent.

a. $x + 8 = 4$	$x + 6 = 2$	$x + 4 = 8$
b. $x + 9 = 0$	$x = 10 = -1$	$x + 10 = 1$
c. $2w + 11 = 5$	$2w = -6$	$2w = 6$
d. $3 = 8 + y$	$5 = 8 - y$	$5 = 10 + y$

G. Equivalent inequalities

1. If two inequalities are such that the solution set of one is the same as the solution set of the other, the inequalities are called equivalent inequalities.

2. Exercise
In each of the following exercises, use the set of all whole numbers as the replacement set and select two inequalities that are equivalent.

a. $y + 3 > 5$	$y > 3 + 5$	$y > 2$
b. $2k \quad 10$	$k > 8$	$k > 5$
c. $r - 2 < 7$	$r < 9$	$r < 5$

H. If the left member and the right member of an equation are operated on by multiplying each by the same non-zero number, an equivalent equation is obtained.

1. Example
$16x = 8$
$\frac{1}{2} \cdot 16x = 8 \cdot \frac{1}{2}$
$1/16 \cdot 16x = 1/16 \cdot 8$

2. Look at $1/16 \cdot 16x = 1/16 \cdot 8$. Is $1/16 \cdot 16x$ another name for x? Is $1/16 \cdot 8$ another name for $\frac{1}{2}$? Then the equation can be written as $x = \frac{1}{2}$. Is the root obvious in this equation? In the equation $1/16 \cdot 16x = 8 \cdot 1/16$ what is the relation between 1/16 and 16? (One is the reciprocal).

3. Suppose we are given the equation $- 2/3r = 5.4$. To obtain a simpler equivalent equation in which the root is obvious, can we operate on the left member and the right member by multiplying each by the reciprocal of the coefficient of the variable? What equivalent equation do you obtain? What is the root of this equation? Of the original equation?

4. To obtain a simpler equation equivalent to $- 2\ 1/3t = - 14$, how will you operate on the left member and the right member? Is the simpler equivalent equation $t = -6$ or is it $t = 6$? What is the root of the original equation?

I. Discovering a second basic principle

1. If the right and left members of an equation are operated on by adding the same number to each or subtracting the same number from each, an equivalent equation is obtained.

2. Exercise (example)

   a. $2x + 8 = 10$
      $2x + 8 - 2 = 10 - 2$
      $2x + 8 - 8 = 10 - 8$

   b. In the equation $2x + 8 = 10$ can we write $2x + 8 - 8$ as $2x$? What is another name for $10 - 8$? Then the equation may be written $2x = 2$. What is the root?

   c. Suppose you are asked to solve the equation $2y + 14 = 4$. How can you use the general rule? You might think: "14 is added to 2y in the left member. How can I obtain an equation that has 2y as its left member? What is the inverse operation of addition? Then you will operate on the left member and the right member by subtracting 14. What equivalent equation do you obtain? $2y = 10$

J. Negative signs may be used to express an operation or a value.

1. Example.

   $5 - y = 17$
   $5 - y - 5 = 17 - 5$ — In these 2 equations "-" is the operation of "y".
   $- y = 12$
   $y = - 12$

2. Exercise and homework in the activities.

K. Solving equations

1. Term: a term may be one of the following
   a. a number -6
   b. a variable x
   c. the product of two terms or two expressions
      $3 \cdot (x+y)$
   d. The quotient of two terms or two expressions x/y

2. Consider the equation $3u + [4 \cdot (-7+13)] = \frac{1}{2} \cdot [9 (-3)]$. To solve this equation, you first express it more simply. What is the simple number that names the product $[4 \cdot (-7+13)]$? (24); the product of $\frac{1}{2} \cdot [9 (-3)]$? (3). Then you obtain the following equation: $3u + 24 = 3$. Continue from here.

3. Homework in the activities.

IV. ACTIVITIES:

A. Class discussion.

B. Mimeographed work.

1. Roots

a. Is there a root of $x+2 = 2\frac{1}{2}$ in the set of positive whole numbers? (No) Does $x+2 = 2\frac{1}{2}$ have a root if the replacement set is the set of all positive numbers? (yes) ($\frac{1}{2}$)

b. Can you find a root of $x + 2 = 2$ in the set of all positive numbers? (no) Can you find a root of $x + 2 = 2$ in the set of non-negative numbers? (yes, 0).

c. Can you find a root of $x + 2 = 1$ in the set of positive numbers? (no) in the set of non-negative numbers? (no) in the set of negative numbers? (yes, -1)

d. Can you find a root of $2x = 5$ in the set of whole numbers? (no) Can you find a root for $2x = 5$ in the set of positive numbers? (yes, $2\frac{1}{2}$).

e. Can you find a root of $4x + 10 = 0$ in the set of positive whole numbers? (no) in the set of all positive numbers? (no) in the set of negative whole numbers? (no) in the set of all negative numbers? (yes)

f. The equation $x^2 = 25$ has two roots in the set of all numbers you know. Are they both in the set of positive numbers? (no) in the set of all whole numbers? (yes, -5, +5)

Using the set of all numbers that you know as the replacement set, find the solution set of each of the following inequalities.

a. $x + 2 < x$	none
b. $x + 1 > x$	all numbers
c. $3y > 3$	all numbers > 1
d. $2y < -4$	all numbers < -2
e. $x + 1 < 2\frac{1}{2}$	all numbers $< 1\frac{1}{2}$
f. $x^2 > 0$	all numbers other than 0

2. Discovering a basic principle.

a. Find the root of each of the following equations.

1. $3u = 12$	4
2. $4r = 17$	$4\frac{1}{4}$

3. - t = 3/2	- $1\frac{1}{2}$
4. - 8x = 24	- 3
5. 3/4k = 12 2/5	16 4/5
6. 8.6 = .43x	20
7. 2s = -20	-10
8. 3.2y = 64	20
9. 7 + 5.2 = 2n	6.1
10. 3 k = 5 - 8 - 6	- 3

3. Discovering a second basic principle. (Solve)

a. m + 3.2 = 6	2.8
b. x - 5 = -2	3
c. 4 + m = 2	- 2
d. x - 2.7 = 3	5.7
e. u + 7 = -2	- 9
f. 3 + r = -2	-5
g. y + 3 = 1	- 2
h. x - $\frac{1}{2}$ = -$\frac{1}{2}$	$\frac{1}{4}$
i. - t + 6 = 1	5

Negative signs

a. 8 - x = 9	-1
b. 4 + (-x) = -4	8
c. - 3 - m = 5	-8
d. m - 1/5 = -1/5	0
e. 2 + y = -2	-4
f. r - 4.3 = -4.3	0

4. Solving equations

a. 5x - 7 3 . (8-2)	5
b. 3y+6 = 5 . (-3)	-7
c. 3m + [2.(-6)] = 4.(-2+1)	2 2/3
d. 6 + $\frac{1}{2}$y = -8 + (-7.4)	-84
e. 1/3M + [3.(-2.2)] = -1.1	16.5
f. $\frac{1}{4}$x - 7 = 3.[1 (-2)]	16
g. 2/3y + [2.(-7 1)] = 4	24
h. 5/7m - 8 = $\frac{1}{2}$.[6 + (-2)]	14
i. 3 1/3 r + 6 = 2.(-3+8)	1 1/5
j. 2 1/5x - [7.(-3)] = -1	-10

APPENDIX E

UNIT V. RATIOS

I. RATIOS AND COMPARISONS

I. OBJECTIVES:

A. To teach and develop a basic understanding of rate and comparison.

B. To teach the students the exact definitions of rate, comparison, and ratio, and how to distinguish among them.

C. To teach the students that the same rate may be expressed by different ratios.

D. To help the pupils learn to distinguish among situations involving fractions, rates, and comparison.

E. To teach the students the ability to solve problems with number pairs.

F. To teach the students the different ways of finding sets of number pairs.

G. To develop the ability to solve problems involving without writing the set.

H. To provide for students of differing abilities.

II. INTRODUCTION:

A. Problem: Rates and Ratios
Mr. Jones bought 12 rosebushes. For all 12 of them he paid $28. The rate at which the rosebushes were sold can be expressed in different ways.

1. One way to express the rate is to use the numerals 12 and 28. You can say that the rosebushes were sold at the rate of 12 bushes for $28.
Here is the way to write 12 and 28 together to express this rate.
12 shows the no. of rosebushes that were sold ___ $\frac{12}{28}$ ___ 28 shows the no. of dollars needed
"12 per 28" -- the word "per" is used in preference to the word "for" when reading a rate.

A pair of numerals used in this way is called a ratio (a pair of numerals written one over the other to express a relation in rate or comparison situations or problems.)
The rate is 12 per 28.
Remember that it takes two numerals to express a rate.

2. We can use other numerals to express the rate at which the rosebushes were sold.
You can see in this picture that the 12 rosebushes and the 28 dollars are separated into 4 equal groups. The dollars in each of these new groups will buy the rosebushes that are in the group.
There are 3 rosebushes and 7 dollars in each group.
Let's think of only 1 of these 4 groups. 3 rosebushes will sell for 7 dollars. (Ask student)
Now you can say that the rate is 3 rosebushes per 7 dollars.
Write the ratio this way:

3 shows the ___	3	7 shows the no.
no. of rosebushes	7 ___	of dollars needed

Now you can say that the rate is 3 rosebushes per $7.
How will the rate be expressed? You can now express the rate as 3 per 7.

You can use other numerals to express the same rate.

$\frac{3}{7}$ $\frac{6}{14}$ $\frac{9}{21}$ $\frac{12}{28}$

Do these two as the others were done.

3. This picture shows more rosebushes and dollars than the other picture did. This time, let's think of the amount of money first. When you think this way, you can express the rate as 35 dollars for 15 rosebushes. Write the ratio in this way:

35 shows the ___	35	15 shows the no.
no. of dollars	15 ___	of rosebushes

How would you express this rate?
35 per 15
Let's look again at the 5 groups. Think of the money first.
You can express the rate as 7 dollars for 3 rosebushes.

How would you write this ratio?

$\frac{7}{3}$

What do each indicate? Point out again these terms:
1. rate
2. ratio

III. SUBJECT MATTER:

A. Terms

1. Rate
2. Ratio
3. Comparison
4. Number pairs

B. Problems

1. (Rate Situation) John bought some apples from the grocery store. He bought 3 apples for 15 cents. (a) How can we express the rate at which the apples were sold? (b) What does the three indicate? The fifteen? In what other ways could this rate be expressed?

2. (Comparison Situation) Paul and Bob were looking at their model planes. Bob had 15 planes to Paul's 10. (a) What is the number pair which compares Bob's planes to Paul's planes? (b) In what other ways could their planes be compared?

3. (Comparison Situation) John and Bob shared their marbles. John had 6 marbles and Bob had 8 marbles. (a) What is the number pair which compares John's marbles to Bob's marbles? (b) What is the number pair which compares Bob's marbles to John's marbles? (c) Does the number pair 3/4 express the same rate?

4. (Comparison Situation) Jim and Tom are sharing marbles. For every 2 marbles Jim takes, Tom takes 1 marble. (a) What number pair compares the marbles Jim takes to the marbles Tom takes? (b) How many marbles does Tom take? (c) What is the number pair that says Jim takes 24 marbles? John takes 12 marbles. (d) Do the number pairs 2/1 and 24/12 tell us the same way of sharing marbles? (We go about this by using the number pairs:)
   2/1, 4/2, 6/3, 8/4, 12/6, 24/12
   Because the number pairs express the same way of sharing, we say they belong to the same set, which can be written in this way:
   2/1, 4/2, 6/3, 8/4, 12/6, 24/12

5. Ann bought some pencils at the school store. She paid 5 cents for the 2 pencils. The price Ann paid for pencils is "5 cents for 2 pencils" or 5/2. (a) What are some other number pairs which show the same price? For 10 cents Ann could have bought 4 pencils. (b) Does 10/4 tell the same price as 5/2? We can write the set of number pairs which

tell the same price as 5/2.
5/2, 10/4, 15/6, 20/8, 25/10, 32/12 ...

6. Mrs. Brown bought 2 cans of applesauce for 21 cents.

   a. What is the set of number pairs which tell the price as "cans of applesauce for cents"?
   b. How much would 6 cans of applesauce cost?
   c. How much would 10 cans of applesauce cost?'
   d. How many cans of applesauce could Mrs. Brown buy for 84 cents?

7. The cub scouts went on a hiking trip. They walked 5 miles in 2 hours.
   a. What is the set of number pairs which show the rate as "miles per hour"?
   b. At the same rate, how far would the scouts have hiked in 6 hours?
   c. How long would it take the scouts to walk 10 miles?
   d. How far would the scouts have hiked in 10 hours?

8. Mary bought 2 candy bars for 5 cents. What would 18 candy bars cost? The number pairs that shows the price "2 candy bars for 5 cents" is 2/5. We want another number pair which shows the price "18 candy bars for ___ cents." Use "n" as a place holder for the number to be found in this pair. Using "n" we would write 18/n. (Why?) Both number pairs tell the same price so we write 2/5 = 18/n. This is called an equation of number pairs. Now think: I know I can find another number pair that tells the same price as 2/5 by multiplying 2 and 5 by the same number.

   $9 \times 2 = 18$

   I must multiply 2 and 5 by 9.

   $$\frac{9 \times 2}{9 \times 5} = \frac{18}{45}$$

   The "n" in the equation can now be replaced by 45.

   $$\frac{2}{5} = \frac{18}{45}$$

   Mary could buy 18 candy bars for 45 cents.

9. Find the number which replaces "n" in this equation. Think: What must I multiply 7 by to make 42?

10. Tony and Bob were comparing their stamp collection. Tony said, "I have 5 stamps for every 6 stamps you have, Bob," Tony has 30 stamps.
    (a) How many stamps does Bob have?
    (b) How do we go about finding the "n"?

11. Mr. White says he can travel 50 miles per hour on a trip. It took him 3 hours to drive from Wausau to Madison.
    a. How many miles is it from Wausau to Madison?
    b. How did you get this number?

12. There are 3 feet in a yard. 21 feet is how many yards?
    a. What is the equation which will solve this problem?
    b. How many yards is 21 feet?

13. Mrs. Stone bought 6 cans of tomatoes for 69 cents.
    a. How much would 2 cans of tomatoes cost?
    b. What is the equation used in this problem?
       $6/69 = 2/n$
    c. The number which replaces "n" is 23 because
       $$\frac{6 \div 3}{69 \div 3} = \frac{2}{23}$$

IV. ACTIVITIES:

A. Introduce the lesson and discuss the introduction with the students to be sure that they understand this process fully.

B. Show the students pictures, using examples of different ratios.

C. Have some of the students explain problems on the board. Ask them questions.

D. Help the pupils who may not understand and are lagging behind.

E. Assign exercises for practice in class and for homework.

1. Exercise 1.

FOR EACH STORY WRITE THE NUMBER PAIRS WHICH SHOW THE COMPARISON RATE, OR THE PRICE.

a. Don bought some apples. The price was 10 cents for 3 apples.

   (1) Write the number pair showing the price as "cents for apples." ________
   (2) Write the number pair that shows the price as "apples for cents." ________

b. Mr. Larson drives 120 miles in 3 hours.
   (1) Write the number pair which tells the rate "miles for hours." ________
   (2) Write the number pair which tells the rate "hours for miles." ________

c. In Miss Jones's fourth grade class there are 12 boys and 14 girls.
   (1) Write the number pair that compares the boys to the girls. ______
   (2) Write the number pair that compares the girls to the boys. ______

d. Bill saw this sign on the side of the road:

SPEED
65
MPH
LIMIT

Bill's father said, "The sign says the speed limit is 65 miles per hour."

   (1) Write the number pair that shows the rate "miles per hour." ______
   (2) Write the number pair that shows the "hours for miles." ______

e. Mary bought some candy bars. She paid 25 cents for 6 candy bars.

   (1) Write a number pair which shows the price "cents for candy bars." ______
   (2) Write a number pair which shows the price "candy bars for cents." ______

2. Exercise II.

FIND AT LEAST 5 OTHER NUMBER PAIRS THAT BELONG IN EACH SET.

a. 3/1 ______ ______ ______ ______ ______

b. 3/4 ______ ______ ______ ______ ______

c. 7/3 ______ ______ ______ ______ ______

d. 1/3 ______ ______ ______ ______ ______

e. 5/6 ______ ______ ______ ______ ______

f. 4/7 _____ _____ _____ _____ _____

g. 2/3 _____ _____ _____ _____ _____

h. 1/2 _____ _____ _____ _____ _____

i. 12/1 _____ _____ _____ _____ _____

3. Exercise III.

FIND THE ANSWERS TO ALL THE QUESTIONS BY FIRST WRITING THE SET OF NUMBER PAIRS WHICH TELL THE STORY. FIND AT LEAST 6 NUMBER PAIRS IN EACH SET.

a. It takes 2 cups of flour to make 3 dozen cookies.
   (1) Write the set of number pairs which shows the rate as "cups of flour for dozen cookies."

   ____

   (2) How many cups of flour would it take to make 9 dozen cookies? ______________________
   (3) 4 cups of flour would make how many cookies?

   ____

   (4) How many cups of flour would it take to make 12 dozen cookies? ______________________

b. Sally saves 5 cents a week. Jane saves 8 cents a week.
   (1) Write the set of number pairs which compare the cents Sally saves to the cents Jane saves.

   ____

   (2) When Sally has saved 20 cents, how much will Jane have saved? ______________________
   (3) How much will Sally have saved when Jane has saved 40 cents? ______________________
   (4) When Sally has saved 35 cents, how much will Jane have saved? ______________________

c. There are 3 feet in 1 yard.
   (1) Write the set of number pairs which shows the rate as "feet for yards." ______________________
   (2) How many feet are there in 5 yards? ____________
   (3) There are 12 feet in how many yards? ____________
   (4) How many feet are there in 6 yards? ____________

d. Bill bought some candy. The price of the candy was two pieces for 1 cent.
   (1) Write the set of number pairs which tell the price as "pieces of candy for cents."

   ____

   (2) How much would 6 pieces of candy cost? ____________
   (3) How much would 10 pieces of candy cost? ____________

(4) How many pieces of candy could Bill buy for 6 cents?

____

4. Exercise IV.

FIND THE NUMBER WHICH REPLACES "N" IN EACH OF THE FOLLOWING EQUATIONS.

a. 2/5 = 8/n

b. 7/9 = 14/n

c. 2/7 = 12/n

d. 1/2 = n/16

e. 7/6 = 28/n

f. 6/100 = 36/n

g. 4/15 = 28/n

h. 35/3 = n/9

i. 6/124 = 18/n

j. 72/7 = n/56

5. Exercise V.

FOR EACH PROBLEM WRITE THE EQUATION WHICH TELLS THE STORY. USE "N" TO HOLD THE PLACE OF THE NUMBER YOU ARE TO FIND. THEN FIND THE NUMBER WHICH REPLACES "N".

a. Mary is making paper hats for a party. It takes her 15 minutes to make 2 hats. How long will it take her to make 10 hats?

______________________________ minutes
15/2 = n/10

____________________ hats

b. Sue bought 2 books for 29 cents. How much would 6 books cost?

______________________________ books
2/29 = 6/n

____________________ cents

c. Mrs. Stone's recipe for custard uses 3 eggs for 2 cups of milk. How many eggs will Mrs. Stone need if she uses 6 cups of milk?

d. 2 bags of marbles cost 25 cents. How much would John pay for 10 bags of marbles?

II. SOLVING PROBLEMS WITH NUMBER PAIRS

I. OBJECTIVES:

A. To develop in the student the ability to apply to test (called the "ratio test") to determine the equality of ratios.

B. To teach the pupils how to use the ratio test to solve equations of ratios.

C. To encourage the students to use mental calculations when paper work is unnecessary.

D. To develop in the students the ability to apply their knowledge of ratios to problem solving in many situations.

E. To develop in the students an appreciation of the concept of ratios as they can be applied to many kinds of problems in the arithmetic of everyday life.

F. To measure the student's accuracy in working ratios.

II. INTRODUCTION;

A. Explanation:
All of the number pairs that you have been working with are called ratios. A ratio is a number pair which tells a price, rate, or comparison. We will call the new way that you will learn to show that two ratios are equal the "ratio test."

B. Problem:
Write this set of number pairs on the blackboard.
2/3, 4/6, 6/9, 8/12, 10/15, 12/18 .....

1. Here is a set of number pairs. We know that any two number pairs in the set are equal because each number pair tells the same price, rate or comparison. 4/6 and 6/9 are both in the set, so we can write
   4/6 = 6/9
   Look at the numbers 4 and 9. Let's multiply 4 x 9 = 36.
   Look at the numbers 6 and 6. Let's multiply 6 x 6 = 36.
   Does 4 x 9 = 6 x 6?

2. Let's look at two other number pairs in the set: 6/9 and 10/15.
   If we multiply the 6 by 15 = 90.
   If we multiply the 9 by 10 = 90.
   Does 6 x 15 = 9 x 10?

3. Look at the number pairs 4/6 and 12/18.
   What is the product of 4 and 18?
   What is the product of 6 and 12?
   What is the product of 6 and 12?
   Now, you should be able to tell if any two number pairs are equal. Does 3/4 equal 39/50?

   $3 \times 50 = 150$
   $4 \times 39 = 156$

   Because the products are not the same, 3/4 does not equal 39/50. We can write this as

   $3/4 \neq 39/50$

   When we draw a line through the equal sign it means "is not equal to."

III. SUBJECT MATTER:

A. Problems.

1. Betty bought some ribbon. She paid 7 cents for 3 yards of ribbon. What would 12 yards of ribbon cost?
   The equation which tells the stody is:

   $7/3 = n/12$

   One way to solve the problem would be to multiply 7 and 3 by 4.

   $$\frac{4 \times 7 = 28}{4 \times 3 = 12}$$

   Another way to solve the problem would be to use the ratio test. Since the two ratios 7/3 and n/12 are equal we know the product of 3 and n must equal the product of 7 and 12. We can write this equation:

   $3 \times n = 7 \times 12$ or $3 \times n = 84$

   To find the number which replaces n we must divide 84 by 3.

   $84 \div 3 = 28.$

   $3 \times 28 = 84$

   Now we know that 28 replaces n in the first equation.

   $7/3 = 28/12$

   Betty would pay 28 cents for 12 yards of ribbon.

2. George can walk 4 blocks in 6 minutes. How far can he walk in 27 minutes?

   $4/6 = n/27$ $\qquad$ $6 \times n = 4 \times 27$

3. Ann bought 5 records for $1.80. How much would 6 records cost?

   5/180 = 6/n   5 x n = 6 x 180

4. On a trip, Mr. Green drove 219 miles in 6 hours. How far did he travel in 4 hours if he traveled at the same rate for the whole trip?

   6/219 = 4/n   6 x n = 4 x 219

5. There are 9 square feet in 1 square yard. 30 square feet is how many square yards?

   9/1 = 30/n   9 x n = 30 x 1

6. Which is the lower price? The set of ratios which show the price 25¢ for 3 cans of corn is

   25/3, 50/6, 75/9, 100/12, 125/15, ...

   or the set of ratios which show the price 33¢ for 4 cans of corn

   33/4, 66/8, 99/12, 132/16, 165/20, ...

   99/12 is less than 100/12

   therefore, 33/4 is less than 25/3

   The price 33¢ for 4 cans of corn is lower than the price 25¢ for 3 cans of corn.
   In order to compare two ratios they must have the same second term.

   33/4 = 99/12   25/3 = 100/12

7. Which price is more, 55 cents for 3 cans of soup or 37 cents for 2 cans of soup?

   55/3, 110/6, 165/9, 220/12, 275/15
   37/2, 74/4, 111/6, 148/8, 185/10

   110/6 is less than 111/6; 55/3 is less than 37/2

8. Which ratio tells the greatest rate?

   4/5 **OR** 2/3 **OR** 13/15

IV. ACTIVITIES:

A. introduce the lesson and discuss the introduction with the students.

B. Help slow students individually.

C. Have students bring in newspaper advertisements showing the use of ratios.

D. Review everything covered in the unit briefly.

E. Assign exercises for practice in class and for homework.

Exercise I

Use the idea you have just learned to tell if the 2 number pairs in each exercise, are equal. If they are equal, write = between them; if they are not equal, write ≠ between them.

1. 7/8	6/7	4. 5/8	65/104
2. 2/6	7/21	5. 7/9	9/15
3. 8/10	12/15		

Find the number pair which does not belong to the set.

1. 5/7, 25/35, 65/84, 80/112, 105/147
2. 6/5, 36/30, 78/65, 102/80, 138/115

Exercise II

For each problem, write the equation of ratios which tells the story. Use n to hold the place of the number you want to find. Then use the ratio test to find the number that replaces n.

1. Mrs. Smith uses 2 cups of flour to make 4 dozen cookies. How many cups of flour must she use to make 6 dozen cookies?

   $2/4 = n/6$
   $4 \times n = 2 \times 6$

2. Mary can buy 6 boxes of candy for 50 cents. She needs 27 boxes of candy. Cost?

   $6/50 = 27/n$
   $6 \times n = 27 \times 50$

3. There are 4 quarts in a gallon. 52 quarts is how many gallons?

Find the number which replaces n in each of the following equations.

1. $4/5 = 64/n$	4. $2/7 = n/91$
2. $6/9 = n/39$	5. $8/20 = 6/n$
3. $4/3 = 56/n$	

Exercise III

Find the number which replaces n in each equation.

1. 1/3 = n/100
2. 5/12 = n/8
3. 15/25 = 21/n
4. 7/30 = n/100
5. 24/15 = 40/n

For each exercise write the equation and find the number which replaces n.

1. There are 9 square feet in 1 square yard. 30 square feet is how many square yards?

2. Joan is writing invitations to a party. It took her 30 minutes to write 5 invitations. She is sending 18 invitations. How many minutes will it take her to write the 18 invitations?

3. At the Acme factory a machine can produce 15 bolts every 12 minutes. How long does it take the machine to make 100 bolts?

Exercise IV

A. Find five more ratios which belong to each of these sets.

1. 12/4,
2. 3/7,
3. 25/100,
4. 6/18,
5. 2/3,
6. 12/1,
7. 8/11,

B. Find the number which replaces n in each of the following exercises.

1. 3/29 = 15/n
2. n/35 = 20/7

3. 15/50 = n/30

4. 8/50 = 12/n

5. 6/21 = 10/n

6. 6/36 = 5/n

C. For each problem, write the equation of ratios which tells the story. Use n to hold the place of the number. Use the ratio test to find the number that replaces n.

1. Mrs. Jones bought 8 cans of corn for 60 cents. How much would 6 cans of corn cost?

2. Mrs. Smith uses 2 cups of flour to make 4 dozen cookies. How many cups of flour must she use to make 6 dozen cookies?

3. The class was serving punch at their party. They knew that 2 gallons of punch served 24 people. They made 9 gallons of punch. How many people could they serve?

4. There are 3 feet in 1 yard. 45 feet is how many yards?

D. Use the ratio test to tell if the 2 ratios in each exercise are equal. If the 2 ratios are equal, write = between them. If the 2 ratios are not equal write ≠ between them.

1. 7/23	84/276
2. 9/4	144/60
3. 12/17	96/136
4. 16/24	40/64
5. 15/6	120/46

III. PER CENT OF INCREASE AND DECREASE

I. OBJECTIVES:

A. To develop the students' powers of analytical thinking.

B. To relate the new (per cent of increase) with that previously learned, namely the second case of percent.

C. To provide a gradual development of the steps involved in solving problems including per cent of increase.

D. To develop understanding of the terms involved: namely, amount of increase, amount of decrease, original amount.

E. To develop the ability to work problems involving increase.

II. **INTRODUCTION:**

We have previously learned three different kinds of per cent problems. We learn how to set up rate pairs and equal ratios. Today we will learn an extension of the second type of per cent problem, which you will remember is the type wherein we find what per cent one number is of another. This extension of this type of problem is finding what per cent of increase we find in certain given situations.

III. **SUBJECT MATTER:**

A. Demonstration Problem

Marjorie is a salesgirl in a dress shop. Her salary now is $60 a week. When she took the position, her salary was $48 a week. What is the per cent of increase?

Let us take this problem and analyze it, naming various parts as we do so. What amount of money did Marjorie start out with?
$48, therefore we shall call this number the Original Amount. The original amount then is the number which was increased in a problem involving increase. Or we might say it is the number which was changed. In a problem involving decrease, the original amount is the number which was decreased, or the number which was changed.

For example, if I say that last week you had 25 cents and this week you have 30 cents, then the original amount is (pause here for students' response) because 25 cents is the number which (pause for students' response) was increased or was changed.

On the other hand if I say last week you had 30 cents and this week you had 25 cents, then the original amount is (30 cents) because 30 cents is the number which (was decreased) or changed.

Notice well that the original amount does not always have to be the larger number or the smaller number. You must read the statement carefully, think whether the problem involves increase or decrease, then select the number which was thus changed.

Now, let us look at our problem. The amount of increase was

$60-$48 or $12

We wish to find the per cent of increase. You remember when we wanted to find what per cent one number was of another, we set up equal ratios, didn't we? For example, if the problem was stated thus:

What per cent of 4 is 5? We set up the problem like this.

$4/5 = X/100$
$5X = 400$
$X = 80\%$

Can we set up our problem similarly? We can say then:

$\$12/\$48 = x/100$
$\$48x = 1200$
$x = 25\%$

By this time many of the students should be ready for short cuts, reducing 12/28 to 1/4 mentally, and changing to 25%.

B. Last summer Ted worked on a farm, earning $12 a week. This summer he will be paid $15 a week. What is the per cent of increase?

Think: The amount of increase is $15-$12 or $3. The increase was $3/12; therefore set up ratios as $3/12 = x/100$.
$12x = 300$
$x = 25\%$

C. The price of oranges increased from 49 cents to 69 cents a dozen. What was the per cent of increase, to the nearest tenth of a per cent?

$69-49-20$
$20/49 = x/100$
$49x = 2000$
$x = 40.8\%$

D. As a holiday season approached, the sales of a gift shop increased from $1800 a month to $2400 a month. What is the per cent of increase?

$2400-$1800 = $600
$600/$1800 = x/100
1800x = 60000
x = 33 1/3%

E. If the price of a loaf of bread is increased from 15 cents to 17 cents, what is the per cent of increase (to the nearest tenth)?

17 - 15 = 2
2/15 = x/100
15x = 200
x = 13.3%

F. Mary bought a bicycle for $27. When she moved, she sold it for $18. What was the per cent of decrease in the price of the bicycle?

Think: The decrease was $27-$18 or $9.
Express the ratio: $9/$27 = x/100
27x = 900
x = 33 1/3%

G. Find the per cent of decrease in each of the following:

Grade	Enrollment last year	Enrollment this year	Per cent of decrease
6	40	35	12 1/2%
7	36	30	16 2/3%
8	32	30	6 1/4%

H. Bill's weight was 111 pounds. After he was 11 he weighed 105 pounds. To the nearest tenth of a per cent what was the per cent of decrease in Bill's weight?

111-105 = 6

6/111 = x/100

Answer: 5.5%

I. Mrs. Nye bought a suit for $49.50 which had formerly been priced at $55. What was the per cent of decrease in the price of the suit?

$55-$49.50 = $5.50
550/5500 = x/100
x = 10%

J. There were 36 pupils in our seventh grade last year. This year there are 32 pupils. What is the per cent of decrease in the number of pupils?

36-32 = 4
4/36 = x/100
x = 11 1/9%

K. A beauty shop advertised: "Regular $6.50 permanent waves now $3.95." To the nearest tenth of a per cent what was the per cent of decrease in the price of the permanent wave?

$6.50 - $3.95 = $2.55
255/650 = x/100
X = 39.2%

IV. ACTIVITIES:

A. Reteach yesterday's problems.

B. Help students correct mistakes.

C. Work several problems with careful guidance.

D. Continue work independently, observing carefully in order to help those who need help.

E. Complete problems as a home assignment if necessary.

. FURTHER PROBLEMS INVOLVING PER CENT OF INCREASE AND DECREASE

I. OBJECTIVES:

A. To develop further the ability to think through the problems, and determine the means of solving them.

B. To provide for the needs of the more rapid students as well as those who are slower.

II. **INTRODUCTION:**

A. We have learned how to find per cent of increase and of decrease. Today we shall develop our thinking a bit more by working some of both types of problems. You must read carefully and determine which number is the original carefully and determine which number is the original amount, because this step is the one most likely to give you difficulty.

B. Before we begin with our new work, are there any questions or difficulties about the work we completed yesterday?

III. **SUBJECT MATTER:**

A. Mr. Bond bought a house for \$8000. Later he sold the house for \$9500. What was the per cent of increase in the price of the house?

$\$9500 - \$8000 = \$1500$
$1500/8000 = x/100$
$X = 18\ 3/4\%$

B. Early in the season peaches cost \$3.60 a bushel. Later the price decreased to \$2.95 a bushel. To the nearest tenth of a per cent find the per cent of decrease in the price of each bushel.

Ans. 18.1%

C. Jim's first test showed that he could read 210 words a minute. The results of the next test showed that he read 240 words a minute. Find the per cent of increase in his reading rate.

Ans. 14 2/7%

D. Mrs. Allen bought a ham that weighed 11 pounds. After it was cooked it weighed 9 pounds. To the nearest tenth of a per cent, find the per cent of decrease in the weight of the ham.

Ans. 18.2%

E. In our graduating class of January there were 36 graduates. What was the per cent of increase in the number of graduates in June? The number of graduates in June was 81.

Ans. 125%

F. One year the winning players on the Yankee base ball team each received $5700 from the World Series games. The next year each player received $5928. Find the per cent of increase in the amount each player received.

Ans. 4%

G. During one summer the membership in the swimming club decreased from 150 to 120. Find the per cent of decrease in the membership of the club.

Ans. 20%

H. In buying shingles for a new roof Mr. Sellers bought shingles that had been $1.85 per bundle for $1.58. To the nearest tenth of a per cent, what was the per cent of decrease in the price of each bundle of shingles?

Ans. 14.6%

I. The new gymnasium at Willard School seats 500 people. The old one had a seating capacity of 275. To the nearest tenth of a per cent, what is the per cent of increase in the seating capacity?

SUPPLEMENTARY WORK

J. When the town of Mill Creek was settled in 1720, the population was only 75 people. Today the population is 90. What per cent has the population increased since 1720?

Ans. 12%

K. In June an automobile factory produced 7500 cars; in July the production dropped to 6375 cars. What per cent smaller was the July production than that in June?

Ans. 15%

L. Shoes that sold for $8.00 a pair 10 years ago are now selling $12.00 a pair. At what per cent more are the shoes selling now than 10 years ago?

M. The population of New City is 12,500 while that of Bedford is 28,000. The population of Bedford is what per cent larger than that of New City?

Ans. 124%

IV. ACTIVITIES:

A. Correcting mistakes on previous day's work.

B. Students will work problems indicated.

V. UNIT TEST

Directions: Solve the following problems. You will be graded on ten of the twelve problems.

1. Explain briefly the following terms:

   A. Commission

   B. Net proceeds

2. Mrs. Bell needs new bath towels. During the January sales, she bought towels that were $1.70 for $1.19. What was the rate of discount?

3. What is the commission on a sale of $6000, if the agent receives $1\frac{1}{2}$% commission?

4. Sue was selling magazine subscriptions. In November she sold 40 subscriptions and in December she sold 64. What was the per cent of increase in the number of subscriptions sold?

5. On a sale of $6000 an agent received $90 as his commission. What rate of commission was he paid?

6. Miss Drew works in a department store. She receives $35 a week and in addition a commission of 3% on her weekly sales. Find her total earnings in a week when her sales amounted to $300; $425; $550; and $785.

7. A sweater priced at $3.40 was on sale at a reduction of 25%. How much was the reduction?

8. At a sale a furniture dealer sold a $150 bedroom suite for $120. Find the per cent of decrease in the price of the bedroom suite.

9. In the last ten years Miss Martin's salary has increased from $1375 to $2040. To the nearest tenth of a per cent, what is the per cent of increase in her salary?

10. Mary found some sweaters that had been priced $2.50 marked 20% off. What the sale price?

11. What must be the amount sold by an agent, if a commission of $1\frac{1}{2}\%$ pays him $90?

12. Find the commission and the net proceeds in each of the following:

   a. $300, 6%

   b. $805, 40%

   c. $4000, 5%

   d. $82.65, 2%

APPENDIX F

UNIT VI. ANALYZING GEOMETRIC FORMS

I. OBJECTIVES:

A. To give the students a simple, practical definition of geometry.

B. To introduce students to the tools of geometry--protractor, compass, straightedge.

C. To develop a basic foundation for understanding by defining certain geometric concepts, and applying these definitions in their most practical sense.

D. To clarify some of the simplest axioms, such as: "Two lines can intersect in only one point," "if two points of a line lie in a plane, all points on the line lie in the plane."

E. To work with students in defining those geometric terms which they will use in their study.

F. To illustrate to students the difference between the area and the perimeter of a plane figure.

G. To give students the basic formulas for finding perimeter, area, and volume for the figures they will be working with in this unit.

H. To provide practice in using these formulas and applying them in familiar, everyday experiences.

I. To develop in the students the ability to identify elements of an angle.

J. To review with students the ways of naming an angle.

K. To review the different kinds of angles and the identifying trait of each.

L. To provide students with a clear understanding of how to measure angles and line segments.

M. To illustrate and provide practice in constructing an angle bisector.

N. To illustrate and provide practice in constructing the bisector of a line segment.

O. To repeat the exercise with the perpendicular bisector of a line segment.

P. To provide students with more than one method of constructing perpendicular bisectors.

Q. To familiarize students with "transversals."

R. To introduce students to the angles formed by a transversal.

II. INTRODUCTION:

Throughout our unit in geometry, there are two ideas that we should constantly keep in mind: (1) that of discovering a result, and (2) that of proving informally that the result is correct in all cases satisfying the conditions of the situation. These two ideas are basic to a mathematician. He discovers a result first by considering particular cases, by measurement, or by making models. Then he attempts to prove this result without the use of particular cases, measurement, or models. It is these two ideas (analysis and deductive proof) that will appear again and again in our subsequent study of mathematics.

From this point the teacher may allow students time to discuss the many ways geometry fits into our daily lives.

III. SUBJECT MATTER:

A. Analyzing Geometric Forms

1. Geometric points and lines

a. A geometric straight line is a set of points extending indefinitely in either direction.

(1) It is an idea.
(2) It cannot be seen.
(3) It is named according to its points.

Example: 

line AB

(4) The set of points in a line may also be used.

Example: 

line x

2. Lines--axioms

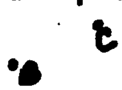
a. There is another point between any two points.

b. There are an unlimited number of lines through a point.

Example: Make a dot to represent a point. Label it A. Can you draw one line through A? (Yes) Two lines? three lines? Any number of lines? (Yes in each case)

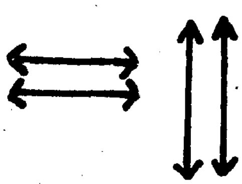
c. There is exactly one line through any two points.

Example: Consider the two points B and C at the left. Could you draw one line that goes through both points? (Yes) two lines? (No)

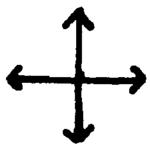


d. Two lines can intersect in only one point.

Example: Draw two lines on your paper. Do the lines cross or <u>intersect</u>? (They may or may not) Do they have to intersect? (No) If the lines you draw intersect, how many points do they have in common? (One) Could the two lines intersect in more than one point? (No)



e. Two lines in the same plane that do not meet are <u>parallel lines</u>.



f. Two lines that intersect to form right angles are perpendicular to each other. Are all horizontal lines in a plane perpendicular to all vertical lines in the same plane? (Yes)

g. Lines that are slanting, not horizontal or vertical, are <u>oblique</u> lines. Draw oblique parallel lines, oblique perpendicular lines.

h. Lines that are neither intersecting nor parallel are <u>skew</u> lines. (Use two pencils to form a model of skew lines.)

3. Line Segments

a. A set of points consisting of two end points and all the points on the line between them (definition).

(1) Named by its end points

Example: 

line segment AB

Could you call it line segment BA? (Yes) How does this diagram differ from the diagram of a line? (It has no arrows.)

(2) Axioms

(a) There are an unlimited number of points between any two points on the line segment.
Example: Draw a line on your paper. Name two points on it M and N. You know there is a point on the line between M and N. Name it P. Is there a point on the line between M and P? (Yes) How many points are between M and N? (An unlimited number.) How many points are on the line? (An unlimited number)

(b) There are an unlimited number of points on a line segment.

4. Planes

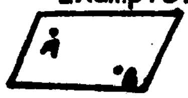
a. A set of points containing an unlimited number of points and extending without limit (definition).

(1) Also containing an unlimited number of lines

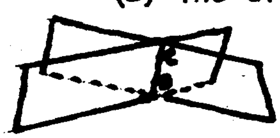
(2) A flat surface (Give examples of planes)

b. Axioms

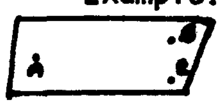
(1) If two points of a line lie in a plane, all the points of the line lie in the plane.
Example: At the left is shown part of a plane containing two points, A and B. How many lines go through A and B? (One) Do all the points of the line AB lie in the plane? (Yes)



(2) The diagram shows two planes that intersect. Do the planes intersect in a line? (Yes) Could they intersect in more than one line? (No) Could they be drawn so that they would not intersect? (Yes)



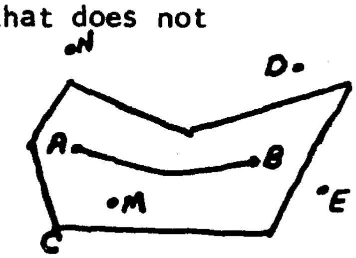
(3) There are an unlimited number of lines in a plane. The lines in a plane and the plane itself are both unlimited in extent.
Example: The representation of a plane at the left has three points marked on it. Are all the points of line AB in the plane? of line CB? Choose another point, D, on AB. Would all the points of CB be in the plane? (Yes in each case) How many planes go through one point? two points? (an unlimited number in each case)



5. Plane figures

a. All the points of the figure lie in the same plane (definition)

b. Think of the set of points that make up a plane as two or more sets of points. Choose two points, A and B, in the same plane as the figure at the right. If you can draw a line or a curve between them that does not intersect the figure, the two points lie in the same part, or region, of the plane. They are members of the same set of points. We say the figure divides the plane into three sets of points: those <u>inside</u> the figure, those <u>on</u> the figure, and those <u>outside</u> the figure.



Simple closed figures are closed figures that have just one inside region and divide the plane in which they lie into three sets of points. Example: 

A line divides a plane into the set of points above the line; the set of points on the line; the set of points below the line.

Segment CD divides the plane into the set of points on the segment and the set of points not on the segment.

6. Polygons

a. Simple closed figures that consist of line segments joined in turn, so that each end point of a segment is an end point of another segment. (Use diagrams of non-regular polygons until the general concept of a polygon is fixed.)

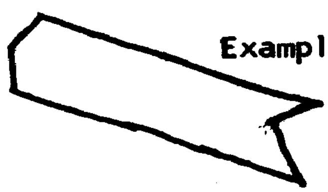
b. The line segments are called the sides of the polygon, and the points at which two line segments join are called vertices (pl. of vertex). (Use example.)

c. If two sides have a vertex in common, they are called adjacent sides. (Use example.)

d. Two vertices that have a common side between them are called adjacent vertices. (Use example.)

e. A line segment joining two non-adjacent vertices of a polygon is called a diagonal. (Use examples.)

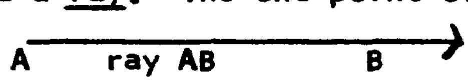
If all the diagonals that can be drawn in a polygon lie inside the polygon, it is called a convex polygon.



Example: Draw all the diagonals you can using the polygon at the left. Is it a convex polygon?

7. Angles

a. Consider line AB. The point A and all points on one side of A are called a ray. The end point of the ray is always named first.



b. The figure formed by two rays with a common end point is called an angle. The ray are the sides, or arms, of the angle.

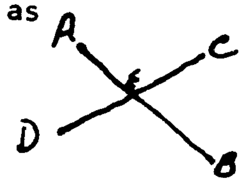
The common end point is the vertex of the angle.

c. Three ways to name angles. (Have practice exercises)

(1) Name three points on it.
(2) Name the vertex and a point on each side.
(3) Name the vertex.

(Sometimes the symbol "∠ " is used to replace the word "angle.")

d. Vertical angles are angles (a pair) such as AED and CEB, with no ray in common.



8. Measuring Angles

The size, or measure, of an angle is the amount of opening between the rays.

a. Size is measured in degrees (^{o}).
b. The protractor, which has 180^{o} marked on it, is used to measure the number of degrees in an angle. (Have exercises using protractor to measure angles.)

9. Measuring Line Segments.

Remember that a line segment is a set of points, and that in dealing with two line segments all the points on one can be matched with all the points on the other.

a. In comparing the opposite sides (line segments) of a rectangle, it would not make sense to write AB=DC, because this would mean that AB and DC are two names for the same line segment. However, we can say "length of AB = length of DC," because this means that the two lengths have the same name, or are equal.

b. Two ways of designating the length of a line segment include:

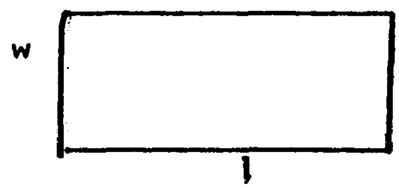
(1) Use "AB" as an abbreviation of both "measure of AB" and "length of AB."

(2) Use a small letter as in the polygon to the right.
mBC = b

10. Perimeters of Polygons

a. The sum of the lengths of its sides, or the distance around the polygon, is called its <u>perimeter</u>.

b. The formula for the rectangle is:
$p = 2l + 2w$ (l representing length; w representing width)
or $p = 2\ (l + w)$ (This is using the Distributive Law)
Work with this formula and with other polygons to obtain other methods for finding perimeter.



11. Circumference of a Circle

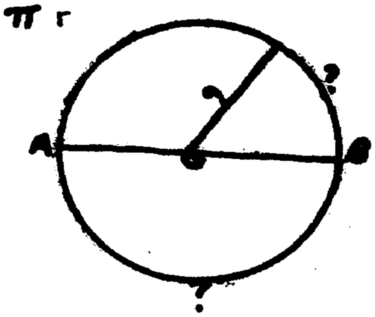
a. A circle is a simple closed figure.

b. A circle is the set of points in a plane at a given distance from a fixed point in the plane called the center. (The center is not a part of the circle.) (Use diagrams on chalkboard to illustrate these points.)

c. The distance around, or length of, a circle is called its <u>circumference</u>.

(1) The quotient of the circumference of a circle divided by its diameter is the same number for all circles -- pi (π)

(2) Formulas for finding the circumference of a circle:
$c = \pi d$ or $c = 2\pi r$



12. Area enclosed by a rectangle

a. A rectangle is formed by line segments.

(1) The inside region, or area, of a rectangle is not a part of the rectangle.

(2) We often say "area of a rectangle" rather than the more cumbersome, but correct "area enclosed by a rectangle."

b. Draw a rectangle. Suppose the rectangle has a length of l units and a width of w units. What is its area? Then could you say that the following is a formula for finding the area enclosed by any rectangle? $A = lw$

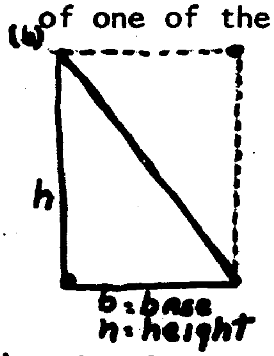
c. Draw a square. Is a square a rectangle? To find the area enclosed by a square, could you use the same formula that you use to find the area enclosed by a rectangle? You know that all the sides of a square are the same length. If we call the length of each side of a square s, is the following a formula for finding the area of any square? (Yes in each case)

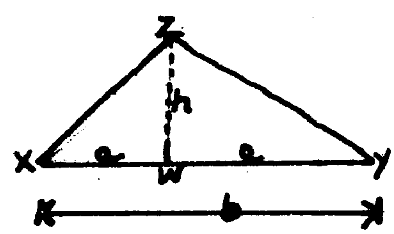
$A = s$

13. Area enclosed by a Triangle

a. A triangle that has a right angle (90°) is called a <u>right triangle</u>.

(1) The symbol "˥" is used to indicate a right angle.
(2) Do the two right triangles at the left form a rectangle? (Yes) Does the sum of the areas of the two triangles equal the area of the rectangle? (Yes) What part of the area of the rectangle is the area of one of the right triangles? ($1\frac{1}{2}$)





(3) Triangle XZY to the right represents any triangle. We can divide it into any two right triangles with line segment ZW by making ZW perpendicular to XY. Is h the height for both right triangle ZXW and right triangle YWZ? (Yes) What letter represents the length of the base of triangle XWZ? of triangle YWZ? (a;c) What is the area of right triangle XWZ? ($\frac{1}{2}ah$) of right triangle YWZ? ($\frac{1}{2}ch$) Is the sum of their areas equal to the area of triangle XZY? (Yes) Then the area of triangle XZY is $\frac{1}{2}ah = \frac{1}{2}ch$ or $\frac{1}{2}h\ (a+c)$.

$A = \frac{1}{2}bh$

From

$$A = \frac{1}{2}ah + \frac{1}{2}ch$$
$$A = \frac{1}{2}h(a + c)$$
But $a + c = b$ $\quad A = \frac{1}{2}hb$
$$A = \frac{1}{2}bh$$

14. Areas Enclosed by Polygons

a. The area enclosed by any polygon can be divided into triangles that do not overlap.

b. The area of the polygon is equal to the sum of the areas of the triangles within it. (Have exercises using various types of polygons.)

15. Area Enclosed by a Circle

a. Just as a rectangle or a triangle encloses an area, so does a circle.

(1) Draw a circle. Inside the circle, draw an octagon.

(a) All of its vertices are on the circle
(b) All of its sides are equal in length

(2) Does $8 \times \frac{1}{2}bh = \frac{1}{2} \times (8 \times b) \times h$? Why? Does $8 \times b$ equal the perimeter of the octagon? (Yes) Is $\frac{1}{2} \times$ perimeter $\times h$ equal to the area of the octagon and close to the area of the circle? (Yes)

b. As the perimeter of the polygon gets closer and closer to the circumference of the circle, the area of the polygon gets closer and closer to being $\frac{1}{2} \times c \times r$ or:
$$\frac{1}{2} \times 2\pi r \times r$$

(1) By performing the indicated operations we finally have a formula for finding the area enclosed by a circle.

$$A = \pi r^2$$

(2) Using circles first, then progressing to other figures with circles or semicircles included, such as a school track area, find the area.

16. Solid Figures that are Polyhedra

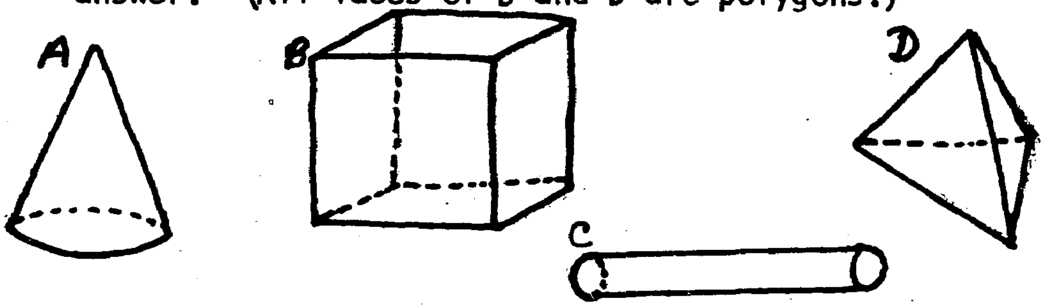
a. If the members of a set of points lie in more than one plane, the set of points is called a figure in space, or a solid figure. (Review definition of a plane figure.)

(1) Open solid figure (Examples: cardboard box)

(2) Closed solid figure (Examples: Pyramids of Egypt; baseball)

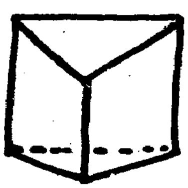
(a) Divides space into three sets of points--the points inside the figure, the points that form the figure, and the points outside the figure.

b. A closed solid figure with all of its sides, or faces, enclosed by polygons is called a polyhedron (pl. polyhedra). Name the figures below that are polyhedra. Explain your answer. (All faces of B and D are polygons.)



c. The intersection of two faces of a polyhedron is called an edge.

An intersection of the edges is called a vertex.



d. A prism is a special kind of polyhedron.

(1) Two faces (called Bases) of a prism are the same size and shape and are parallel.

(2) A prism gets its name from the shape of its base.

(a) The bases are connected by faces that are enclosed by parallelograms.

(b) The sum of the areas of the lateral faces is called the lateral area of the prism.

(3) Give examples and discuss oblique prisms, right prisms, triangular prisms, right rectangular prisms (or rectangular solids), cubes.

e. The polyhedron at the left is a <u>pyramid</u>. What is the name of the lateral faces of a pyramid? (Triangle) Does this pyramid have a base? (Yes) How many vertices does it have? (5) Since the base is a square, it is called a square pyramid. Could a pyramid have a rectangular for a base? a triangle? a hexagon? (Yes for each) Write the special name for each of these types of pyramids. (Rectangular, triangular, and hexagonal)

(1) A pyramid that is a triangular pyramid is called a <u>tetrahedron</u>.

(2) A polyhedron with 12 faces is called a <u>dodecahedron</u>.

17. Volume and Capacity

a. The capacity of a container is the amount that container will hold.

b. To measure the space or bulk, bounded by any closed solid figure is to find its <u>volume</u>.

(1) Volume is measured in terms of the cubic inch, cubic foot, cubic yard, etc.

c. Draw a diagram which shows 1 cubic yard. How many cubic feet "pave" the base? (9) How many layers are there? (3) Then how many cubic feet are there in a cubic yard? (27)

d. If the length of a rectangular prism is represented by l and the width by w, what is the area of the base? lw. If the height is represented by h, what is the volume? lwh. Could a formula for finding the volume of a rectangular prism be written as follows if V represents the volume? (Yes)

$$V = lwh$$

18. Volume of a Prism

a. If the length of a rectangular prism is represented by l and the width by w, what is the area of the base? lw. If the height is represented by h, what is the volume? lwh. Could a formula for finding the volume of a rectangular prism be written as follows if V represents the volume? (Yes)

$$V = lwh$$

If B represents the area of the base (lw), could the formula also be written as V=bh? (Yes)

b. Is a cube a rectangular prism? (Yes) Then what is the volume of the cube with 4-inch dimensions? (64 cu.in.) Write a formula for finding the volume of a cube if the length of an edge of the cube is represented by e.

$$V = e^3$$

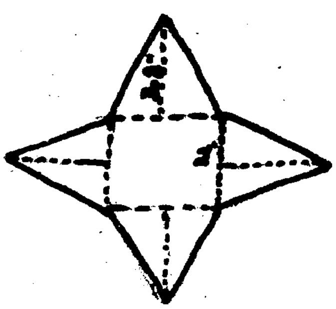
c. If B represents the area of the base of a prism (lw) and h equals the height, the volume of any prism can be found by using the formula: $V = Bh$

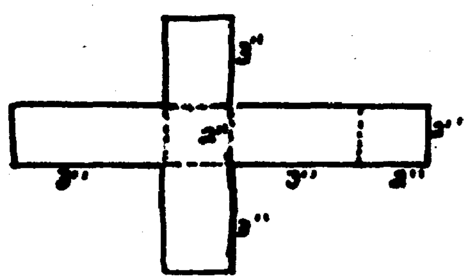
(Giving the dimensions, have students find the volumes of the various types of prisms which are more common or familiar to them.)

19. Volume of a Pyramid

. The following is a formula for finding the volume of any pyramid. $V = 1/3Bh$

The B represents the area of the base, and the h the height from the upper vertex to the base.
To convince yourself that the formula is correct, try the following experiment. Carefully draw diagrams on heavy paper like those drawn here. Cut them out and fold along the dotted lines. Fasten the edges together with tape. If you have made them correctly, you have a square pyramid and a square prism with equal bases and equal heights. Now cut out the top of the prism and the base of the pyramid. Fill the pyramid to the top with sand. Pour it into the prism. Repeat this until the prism is exactly full. How many times did you fill the pyramid? (3 times) Is the capacity of the prism 3 times that of the pyramid? (Yes) Does it seem reasonable to say that the volume of a pyramid is one-third that of a prism with an equal base and an equal height? (Yes)





20. Area of a Cylinder

a. A <u>cylinder</u> is a closed solid figure that has two bases with the same shape.

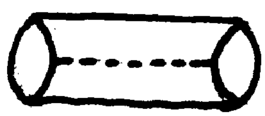
(1) What is the shape of the base? (Circular)

(a) The curved surface of the cylinder is called its <u>lateral surface</u>.

(2) Does a cylinder have sides? (No)

b. The diagram at the right is a cylinder with the bases cut out. Imagine that a cut is made along the dotted line and that the lateral surface of the cylinder is rolled out flat. What is the name of the resulting plane figure? (Rectangle) Is the base of this rectangle the same length as the circumference of the cylinder base? (Yes) Is the height of the rectangle equal to the height of the cylinder? (Yes) What is the area of the rectangle? (ch) What is the circumference of the base of the cylinder? 2wr. Then if S represents the lateral area of the cylinder, we have the following formula.

$$S = 2\pi rh$$



c. What is the area of one base of a cylinder with a radius of r? Do you see that the <u>total area</u> of a cylinder would be twice the area of one base plus the lateral area? If we represent the total area by T, is the following formula true? (Yes)

$$T = 2\pi r^2 + 2\pi rh$$

21. Volume of a Cylinder

a. Draw a cylinder, which has a base with a radius of r, and a height of h. You have seen that as the number of sides of a polygon increases, the area closely resembles that of a circle. Imagine a prism whose base is a many-sided polygon. Do you see that the volume of the prism will closely approximate the volume of a cylinder of equal height? You know that the volume of a prism is Bh. What is B, the area of the base of the cylinder? πr^2 Then is the following formula for the volume of a cylinder?

$$V = \pi r^2 h$$

b. Examples:

(1) A cylindrical wastebasket has a base with a diameter of 9 inches and a height of 14 inches. What is its volume?

(2) The outside dimensions of a silo are: height, 44 ft.; diameter, 20 feet. If the floor and wall are 1 foot thick, what is the volume of the interior of the silo?

22. Volume of a Cone

a. Draw a cone. What is the shape of the base of a cone? (Circular) Does a cone have an upper base? (No)

(1) The point is called the vertex, or apex, of the cone.

(a) How does a cone differ from a cylinder? (It has only one base)

(b) Does a cone have sides? (No)

(2) The curved surface is called its lateral surface.

b. Imagine you had a cone in one hand a cylinder in the other. These two shared a common diameter and a common height. How many times do you think you would have to fill the cone with sand to fill the cylinder? (3 times) Obtain a conical paper cup, construct a cylinder of the same base and height, and test your answer.

The formula for finding the volume of a cone whose base has a radius of r and whose height is h is:

$$V = 1/3\pi r^2 h$$

c. What are the similarities between the formulas for the volume of a cone and the volume of a pyramid? (Both are 1/3 of the area of the base X height.)

d. Examples:

(1) A cone and a cylinder have equal radii. How many times the height of the cylinder must the height of the cone in order that the volume of the cone will be twice that of the cylinder? (6 times)

(2) One cone has a height twice that of another cone. What is the ratio of their volumes if their bases have equal radii? (2:1)

23. Surface of a Sphere

a. Imagine that you have a ball in your hand. Does this sphere (ball) have sides? (No) Does it have any bases? (No) If you cut the sphere into two equal parts what would be the shape of the figure formed by the cut? (a circle) One-half of a sphere is a <u>hemisphere</u>.

b. Using this hemisphere what is the area of the flat surface? (πr^2) Then what is the area of the curved surface? ($2\pi r^2$) Then what is the area of the sphere? ($4\pi r^2$) We state the formula as follows:

$$S = 4\pi r^2$$

24. Volume of a Sphere

Using a hollow rubber ball and a cylinder whose diameter and height are equal to diameter of ball and sand, explain why the volume of a sphere is $V=2/3(\pi r^2 h)$ or $V=4/3\pi r^2$

B. Plane Geometry

1. Classification of Angles

a. If two rays that form an angle extend in opposite directions along a line, the angle is a <u>straight angle</u>.

What is the measure of a straight angle in degrees? 180°

b. One-half of the measure of a straight angle is the measure of a <u>right angle</u>.

What is the measure of a right angle in degrees? 90°

c. If two rays with a common end point extend in the same direction along a line, an <u>angle of 0°</u> is formed.

d. An angle with a measure greater than 0° and less than 90° is called an <u>acute angle</u>.

e. If the measure of an angle is between 90° and 180°, the angle is called an <u>obtuse angle</u>.

(Ask students to draw different types on the chalkboard and let other students describe them.)

f. Tell whether each statement below is true, or false.

(1) If two angles are acute, their measures must be equal. F
(2) If an angle measures 45°, it is acute. T
(3) If an angle is acute, it has a measure of 45°. T
(4) If one angle is acute and a second is obtuse, the measure of the second is larger than that of the first. T
(5) If an angle has a measure twice that of an acute angle, it must be obtuse. F
(6) If an angle has a measure one-half that of an obtuse angle, it must be acute. T
(Have students obtain in their answers individually and then compare them.)

g. In the following exercises, let the replacement set for each exercise be the set of all the angle measures you know.

(1) If x represents the measure of a right angle, what is its solution set? (90°) What is the solution set if x represents a straight angle? (180°)
(2) Suppose x represents the measure of an angle. What is its solution set if 2x is to represent the measure of an obtuse angle? $45^\circ < x < 90^\circ$
If 3x is to represent the measure of an obtuse angle?
$30^\circ < x < 60^\circ$
(3) Draw a straight angle on your paper. Use your protractor to divide it into three angles having equal measures; into four angles of the same size; five angles.

2. Complementary Angles

a. A pair of angles are called <u>complementary angles</u> if the sum of their measures is 90°.

(1) Each angle in a pair of complementary angles is called the <u>complement</u> of the other.

b. You can use equations to solve problems involving complementary angles. For example, find the measures of two complementary angles if one is 20° more than the other.

$$\begin{aligned} x &= \text{the measure of the smaller angle} \\ x + 20 &= \text{the measure of the larger angle} \\ x + (x + 20) &= 90 \end{aligned}$$

c. Find the measures of each of the pairs of complementary angles whose measures have the following relationships.
   (1) They are equal. 45°; 45°
   (2) One is twice the other, 60°; 30°
   (3) One is 10° more than the other. 50°; 40°
   (4) One is 15° less than the other. $37\frac{1}{2}^\circ$; $52\frac{1}{2}^\circ$
   (5) One is 15° more than twice the other. 65°; 25°

3. Supplementary Angles

a. When the sum of the measures of two angles is 180°, they are called <u>supplementary angles</u>.

   Each angle is called the <u>supplement</u> of the other.

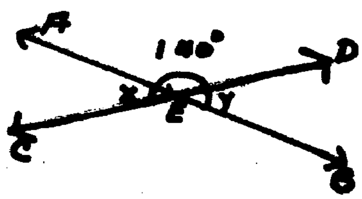
b. Find two supplementary angles whose measures have these ratios.

   3:2 (109°; 72°)   (2) 2:7 (40°; 140°)   (3) 1/2:1/5 ($128\,4/7^\circ$; $51\,3/7^\circ$)

c. Two angles with a common vertex and a common ray that is between their other rays are called <u>adjacent angles</u>.

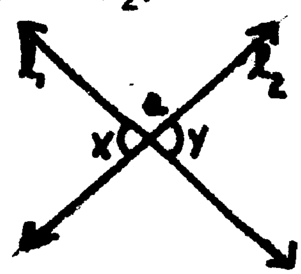
   Two adjacent angles whose exterior sides lie along a line are supplementary.

d. In the diagram at the right, AEB and CED are lines. The measures of two angles formed are x and y. Are angles AED and DEB adjacent? Are they supplementary? Why? (Yes for both; they are adjacent angles whose exterior sides lie along a line.)



   Then what does y equal? (40°) Are angles AEC and AED adjacent? (Yes) Are they supplementary? (Yes) Then what does x equal? (40°) Does y = x? (Yes)

e. Do the intersecting lines at the right, l_1 and l_2, form pairs of adjacent angles? (Yes) Do they form pairs of vertical angles? (Yes) Notice that the measures of three of the angles are a, x, and y. What is the sum of a and x? 180°. Could you represent x as 180°-a? (Yes) How could you represent y? 180°-a. Then are x and y equal? Is this true no matter what replacement is made for x? Then is the statement true for all pairs of vertical angles? (yes in each case). You have proved the statement: If two lines intersect, the vertical angles formed have equal measures.



f. Is there an angle such that the measure of its complement equals that of its supplement? (No)
(For any angle with measure x, complement is represented by $90^{\circ} - x$; supplement is represented by $180^{\circ} - x$; and the equation $180 - x - 90 - x$ has no root.)

4. Bisecting Angles

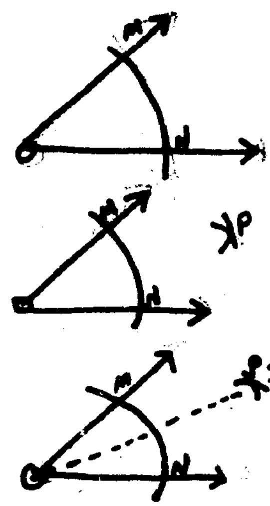
a. To bisect an angle is to divide it into two equal parts; that is, the two parts have equal measures.

Between the rays forming angle is a third ray that bisects it called the <u>bisector</u> or <u>angle bisector</u>. (Draw an angle and use it to illustrate this point and the others that follow it.)

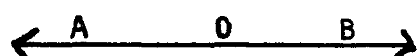
(1) One way to find the bisector of an angle is to use the protractor. (Draw angles and bisect them.)

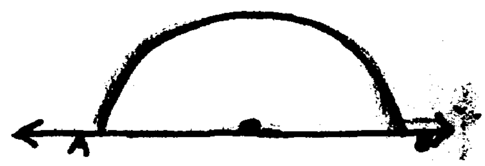
(2) Another way to draw the bisector of an angle is to use compass and straightedge, or ruler. (When we use this method we say we have "constructed" the bisector.)
(Make sure students understand the difference between drawing a bisector and constructing a bisector.)

b. Draw an acute angle, O. Now draw an arc that intersects the sides of the angle in points M and N. Is mOM equal to mON? Why? (Yes, all radii of the same circle are equal.)
Now open your compass to a distance that is greater than one-half of the length of line segment MN. Using M and N in turn as centers, mark arcs that intersect. The point of intersection is labeled P in the diagram. Does mNp equal mmNP? Why? Is Op the bisector? (Yes)

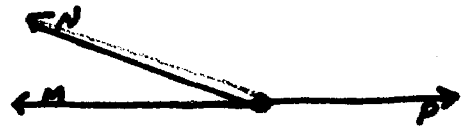


c. At the right is straight angle AOB. Use your protractor to find its bisector.





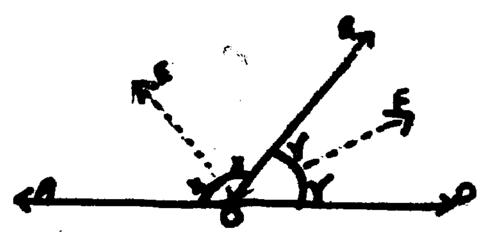
Now repeat the exercise, but this time use compass and straightedge. What kind of angle is each of the two equal parts? Right angle.



d. Draw a pair of adjacent supplementary angles, such as those in the diagram. Draw the bisector of each angle. What is the measure of the angle formed by the two bisectors? 90°

e. Draw another pair of adjacent supplementary angles. Bisect each of the adjacent angles with compass and straightedge. Measure the angle between the bisectors. What is its size? 90°

f. The diagram shows adjacent supplementary angles with their bisectors, OE and OF. The measures of angles AOB and BOD are represented by 2x and 2y. How may you 180°? (Yes) What does x + y (x; equal? (90°) Then what does 180°? (Yes) What does x + y equal? (90°) Then what does m∠EOB + m∠BOF equal? (90°)



<u>The bisectors of adjacent supplementary angles form a right angle</u>.

g. Make a large diagram of a triangle on a sheet of paper. Bisect each angle of the triangle. What do you find? (The bisectors intersect in a point.) Try to make a general statement about the results of your experiment.

The bisectors of the angles of a triangle intersect in a point.

5. Bisecting Line Segments

a. The midpoint of a line segment is the point on the line which is exactly midway between the two end points.

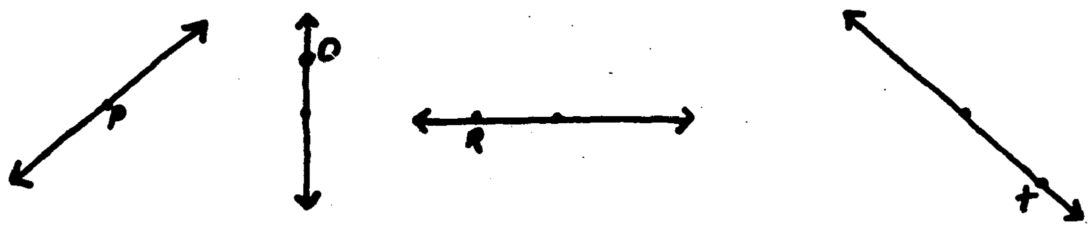
On your paper, draw a line segment 3 inches long. Using your ruler, locate its midpoint.

b. In the above exercise, you have used your ruler to find the midpoint of a line segment. There is a method of finding the midpoint of a line segment using compass and straightedge. Draw line segment AB. With an opening of your compass that is larger than one-half the length of AB, and using A and B in turn as a center, make arcs that intersect above and below AB. What would happen if you had a compass opening that was less than one-half mAB? (Arcs would not intersect.) The points of intersection of the arcs are marked P and Q in the diagram. Draw PQ. Is every point on PQ an equal distance from both A and B? Explain. (Yes. All points of intersection of arcs made with same radius from A and B will fall on line PQ.) Then the intersection of PQ and AB is the midpoint, M, of AB.

6. Perpendicular Lines

a. Two lines which intersect to form right angles are perpendicular lines.

(1) Draw lines in the positions shown below. Now use your protractor to draw a line perpendicular to each line at the labeled point.



(2) Draw a line segment. At its midpoint draw a perpendicular line.
A perpendicular line at the midpoint of a line segment is called the perpendicular bisector of the line segment.

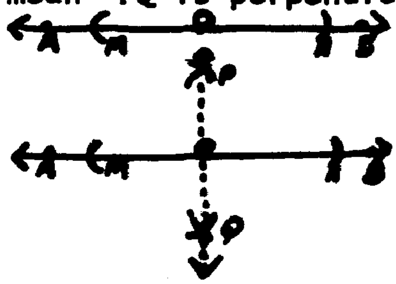
Draw a large triangle with all of its angles acute. Draw the perpendicular bisectors of the sides. Do they meet in a point? (Yes) What results did the other students find? Make a general statement about the perpendicular bisectors of the sides of a triangle.

The perpendicular bisectors of the sides of a triangle meet in a point.

b. Ways of constructing perpendicular bisectors

(1) When you found the midpoint of a line segment with compass and straightedge, you actually constructed the perpendicular bisector of the line segment.

(2) You can construct a perpendicular to the line AB at O if in some way you can mark a line segment on AB that has O for its midpoint. Open your compass to any convenient distance, use O as a center, and make arcs intersecting AB. Is line segment MN on AB? (Yes) Is O its midpoint? (Yes) Why? (OM and ON are radii of a circle with center at O.) We can write "PQ AB" to mean "PQ is perpendicular to AB."



(a) Draw line l on your paper. Construct a line, l_2 so that $l_1 \perp l_2$. Now construct a third line, l_3 so that $l_3 \perp l_2$. Is $L_2 \perp l_1$? What do you think the relationship is between l_3 and l_1? They are parallel.

(b) Another kind of problem is the construction of a perpendicular to a line from a point not on the line. Given line l and point P, as in the diagram, the first step is to obtain a line segment on l that will have P on its perpendicular bisector. You know that if P is on the perpendicular bisector it will be an equal distance from the end points of the needed line segment. Then what instrument will help you mark the end points, M and N, of the line segment? (Compass) You have already arranged for P to lie on the perpendicular bisector of MN. Now construct the perpendicular bisector as shown in the diagram. Is $PQ \perp l$? (Yes)

7. Angles Formed by a Transversal

a. A line that intersects two other lines is a <u>transversal</u>. Draw a transversal.

(1) Pairs of angles which are on the same side of the transversal are <u>corresponding angles</u>.

(2) Angles on opposite sides of the transversal and within the two lines (not using the transversal as a line) are called <u>alternate interior angles</u>. Point these out.

(3) Angles on the same side of the transversal and between the other two lines are <u>interior</u> <u>angles</u> on the same side of the transversal.

(4) Angles that are on opposite sides of the transversal and exterior to the other lines are alternate exterior angles.

b. We may write "$\angle a$" to mean "the angle whose measure is a." Use this method of naming angles to name all the pairs: corresponding angles; alternate interior angles; alternate exterior angles; interior angles on the same side of the transversal.

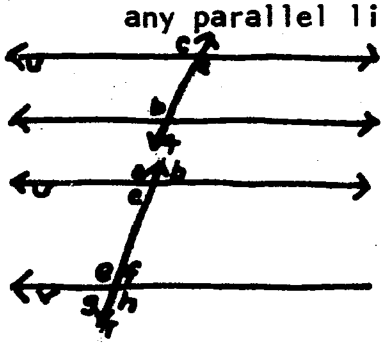
8. Properties of Parallel Lines

a. If two lines in the same plane do not intersect, then the lines are parallel. If two lines are parallel, then they do not intersect.

b. Draw the parallel lines u and v, and their transversal, t. We can write "$u \parallel v$" to stand for "u is parallel to v." Measure one pair of alternate interior angles. Measure the other pair. Do you find that the alternate interior angles have the same measure? (Yes)

c. In the same way, if two parallel lines are intersected by a transversal the measures of pairs of corresponding angles are equal.

d. Let us assume the general principle: If two parallel lines are cut by a transversal, then the alternate interior angles have equal measures.
You have discovered that if two parallel lines are cut by a transversal the measures of corresponding angles are equal. Now let us see if we can prove this statement for any parallel lines and any pair of corresponding angles.



In the diagram, u v and t is a transversal intersecting u and v. Are the angles whose measures are represented by a and b alternate interior angles? Then does a=b? (Yes) Why? (Alternate interior angles are equal.) Are the angles whose measures are indicated by a and c vertical angles? (Yes) Then what can you say about a and c? (They are equal.) We now have a=b and a=c. Can one name be substituted for another namefor the same thing? Then is b=c? Have we proved the following: Corresponding angles have equal measures when two parallel lines are cut by a transversal? (Yes in each)

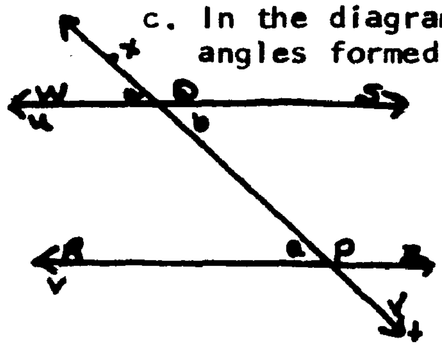
e. Measure each angle in the diagram. What did you find about the measures of ∠a and ∠h? of ∠b and ∠g? Do you think the following statement is true: If two parallel lines are cut by a transversal, then the measures of alternate exterior angles are equal? What is the sum of angles c and e? Is the following a true statement: If two parallel lines are cut by a transversal, then the sum of the measures of interior angles on the same side of the transversal equals 180°? (Yes)

f. You now know several facts about parallel lines with a transversal: The measures of alternate interior angles, corresponding angles, and alternate exterior angles are equal; the interior angles on the same side of the transversal are supplementary.

9. Proving Lines Parallel

a. A general principle is: If two lines are cut by a transversal so that the measures of a pair of alternate interior angles are equal, the lines are parallel. Construct this situation.

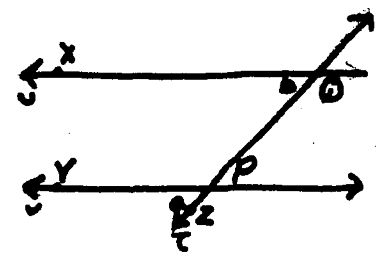
b. Write the converse of this statement. Is the converse a true statement? Was it already known to you? (Yes in each case.) If two parallel lines are cut by a transversal, then the alternate interior angles have equal measures.)



c. In the diagram at the left, a pair of corresponding angles formed by lines u, v, and transversal t have measures of a. Do you think u∥v? (Yes) What is the name of the pair of angles XQW and SQP? (Vertical angles) What can you say about their measures, a and b? (They are equal.) Then does m∠RPQ = m∠SQP? (Yes) What principle do you know about such angles? (If the measures of alternate interior angles are equal, the lines are parallel.) Then is u∥v? (Yes)

You have proved the statement: If two lines are cut by a transversal so that a pair of corresponding angles have equal measures, the lines are parallel.

d. State the converse of the above statement. Is it also a true statement known to you? (Yes) (If two parallel lines are cut by a transversal, then the corresponding angles have equal measures.)



e. The diagram shows two lines cut by a transversal with a pair of alternate exterior angles having equal measures. Let us see if we can prove that the lines u and v are parallel under these conditions. What is the name of the angle pair $\angle b$ and $\angle a$? What can you conclude about their measures? (They are equal.) What name do you give to the pair of angles $\angle XQP$ and YPZ? (Corresponding angles) Since a = b, does $m\angle XQP = m\angle YPZ$? (Yes) Then what principle permits you to say that $u \parallel v$?

(If the measures of corresponding angles are equal, the lines are parallel.

You have proved the statement: If two lines are cut by a transversal so that a pair of alternate exterior angles have equal measures, the lines are parallel.

f. State the converse of this. Is it also true? (Yes) (If two parallel lines are cut by a transversal, then the alternate exterior angles have equal measures.)

g. Suppose you are given two lines cut by a transversal so that a pair of interior angles on the same side of the transversal are supplementary. Let us see if in this case we can prove lines u and v parallel. Are $\angle RQP$ and $\angle SQP$ adjacent angles? (Yes) Do their sides lie along the same line? (Yes) Then does $a + c = 180^\circ$? (Yes) Why? (Two adjacent angles whose exterior sides lie along a line are supplementary.) Is $180^\circ - a$ another name for c? (Yes) We started out with $\angle XPQ$ and $\angle SQP$ supplementary; that is, $a + b = 180^\circ$. Is $180^\circ - a$ another name for b? (Yes) Then what do you conclude about b and c, the measures of alternate exterior angles $\angle RQP$ and $\angle XPQ$? (They are equal.) What statement do you know that fits this situation? (If the measures of alternate interior angles are equal, the lines are parallel.)

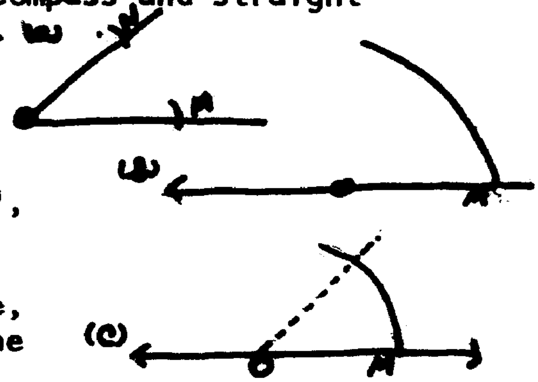
You have proved the statement: If two lines are cut by a transversal so that a pair of interior angles on the same side of the transversal are supplementary, the lines are parallel.

h. State the converse of the above statement. Is it also a true statement? (Yes)
(If two lines are parallel and cut by a transversal, then the interior angles on the same side of the transversal are supplementary.)

10. Drawing a parallel line

a. Suppose we are given a line, l, and a point, P, not on the line. Let us try to draw a parallel line to l through P. Draw any transversal through P that intersects l. Measure one of the angles at Q. Then make a corresponding angle at P with the same measure. Draw the line on which the ray PR lies. What general statement assures you that the line you draw is parallel to l? (If two lines are cut by a transversal so that a pair of corresponding angles have equal measures, the lines are parallel.)

b. A similar process can be used to draw a line parallel to l with compass and straightedge. First, however, you must know how to copy an angle with compass and straightedge. Copy $\angle O$ with your protractor. With your compass open to any convenient distance, use O as a center and make an arc intersecting the sides of the angle at N and M. Now draw a line and choose a point, O', as the vertex of the copied angle. With O' as the center and with the same opening of the compass as before, make an arc and label M'. To find the size of $\angle O$, measure the distance MN with your compass. Now use M' as a center and make an arc intersecting the first arc you draw. Label the intersection N'. Then O'N' is the ray that gives the copied angle, $\angle O$, whose measure is equal to that of $\angle O$.



11. Classification of Triangles

a. Definitions

(1) A triangle is a particular type of polygon--one with three sides. A figure is a triangle if it consists

of three points not on the same line and three line segments joining the points in pairs.

b. Types

(1) If a triangle has two sides the same length, it is called an isosceles triangle.

(2) If a triangle has three sides the same length, it is called an equilateral triangle.

(3) When no two sides of a triangle are the same length, the triangle is called a scalene triangle.

c. Construction

(1) To construct an isosceles triangle, start with a line segment, AB, as the base. Choose an opening for your compass and use each end point of the line segment to mark arcs that intersect. Label the intersection C. Draw AC and BC to form a triangle. Why are two sides of the triangle equal in length? (AC and BC are radii of equal circles.)

(2) To construct an equilateral triangle, you must make all the sides of the triangle the same length. Use your compass to measure the length of the line segment that is to be the base. Then, using each end point of the base, mark arcs that intersect. Connect the point of intersection with the end points. Why are all the sides the same length? (DE, DF, and EF are radii of equal circles.)

(3) The height or altitude of a triangle is the perpendicular from a vertex to the opposite side.

(a) How many altitudes does a triangle have? (3) Construct an isosceles triangle on your paper. With compass and straightedge, construct all the altitudes of the triangle by the method you used to construct a perpendicular from a point to a line.

Did the altitudes intersect in a point? Was the point inside the triangle? See whether you can make a general statement concerning the altitudes of an isosceles triangle. (Altitudes of isosceles triangle intersect in a point inside the triangle.)

(b) Draw a scalene triangle one of whose angles is an obtuse angle. Construct its altitudes. Where do they intersect? (Outside the triangle) How can you modify the general statement made in the (a) part in order to make it apply to scalene triangles as well as isosceles triangles? (The altitudes of a scalene or isosceles triangle intersect in a point inside or outside the triangle.)

(c) Draw right triangle. Construct its altitudes. Where do the altitudes intersect? (At the vertex of the right angle.)

12. The Sides of a Triangle.

a. Draw two triangles having different sizes and shapes. Compare the sum of two sides to the size of the other side in each case. What do you find? (For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.)

b. Draw a triangle whose sides measure 3 inches, $2\frac{1}{2}$ inches, and 1 3/4 inches. Use your protractor to measure angles A and C. Which measure is longer or larger? Which angle lies opposite the longer side? Do you agree that the following principle is true? (Yes)

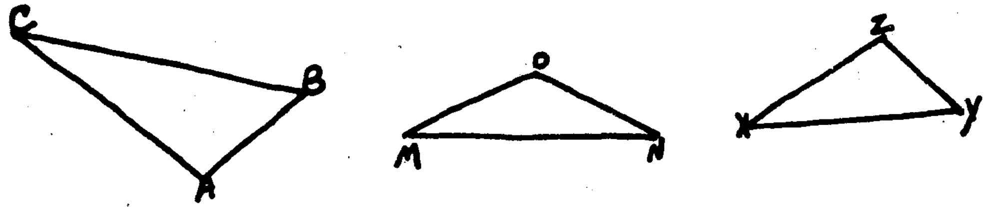
If the measure of one side of a triangle is greater than a second, the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

c. Write the converse of the statement above. Is it a true statement? (Yes)
(If the measure of one angle of a triangle is greater than that of a second, the side opposite the angle with the greater measure is larger than the side opposite the angle with the smaller measure.)

13. Congruent Triangles

a. The diagrams below represent triangles that are exactly the same size and shape. Do you see that one triangle could be moved so that its vertices would fall on, or correspond to, the vertices of another triangle? (The vertices of one triangle may correspond to the vertices of a second triangle in many ways.)

(1) When the vertices correspond, does mAB=mOM? mBC=mMN? mCA=mNO? Then do the respective sides correspond? Does$\angle$A=m$\angle$O? m$\angle$B=m$\angle$M? m$\angle$C=m$\angle$N? (Yes in each case)



(2) If the vertices of two triangles can be made to correspond so that the lengths of corresponding sides are equal and the measures of corresponding angles are equal, the triangles are said to be <u>congruent</u>.

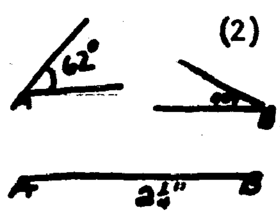
(a) Write the converse of this.
(If two triangles are congruent, the vertices of the triangles can be made to correspond so that the lengths of corresponding sides are equal and the measures of corresponding angles are equal.)

(b) A convenient symbol for "triangle" is "   " and the symbol for "is congruent to" is "   ".

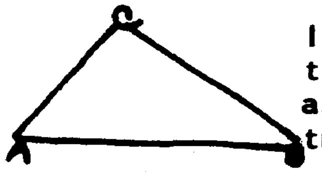
b. Conditions necessary for two triangles to be congruent

(1) On your paper, draw two line segments--AB, 2 inches long, and AC, 3 inches long--with an included angle, A, measuring 75°. Now see if you can construct a triangle, ABC, with these given parts. Is the triangle the same size and shape as the triangles which others draw? Are all the triangles congruent? Do you agree that the following principle is true? (Yes in each case)

If two sides and the included angle of one triangle have the same measures as two sides and the included angle of another triangle, the triangles are congruent.



(2) Shown here are angles A and B and their measures. Line segment AB has a length of $2\frac{1}{4}$ inches. Can you construct a triangle, ABC, with these given parts? The first step in an attempt to construct the triangle is to draw a line segment of $2\frac{1}{4}$ inches. Label the line segment AB. Now construct at A and B angles of the proper size. The sides of angles A and B will intersect at C. You see that:



If two angles and the included side of one triangle have the same measure as two angles and the included side of another triangle, the triangles are congruent.

(3) To discover the third principle of congruent triangles, begin with the lengths of three given line segments; construct a triangle with these measurements as the lengths of its sides. In comparing it with the triangles of others you see that:

If the three sides of one triangle have the same measures as the three sides of another, the triangles are congruent.

c. Have exercises with triangles using these basic principles to prove triangles congruent.

14. Isosceles and Equilateral Triangles

a. Shown at the left is isosceles triangle ABC.
What must be true of two of its sides? Their lengths are equal.)
We call the angle formed by the sides of equal length the <u>vertex angle</u>.

If two sides of a triangle are the same length, the angles opposite these sides have the same measure.

b. What is true of the sides of an equilateral triangle? (They are all the same length.) Is an equilateral triangle an isosceles triangle? (Yes) Apply the statement above to an equilateral triangle, to prove that an equilateral triangle is equiangular.

15. The Quadrilateral Family

a. Have diagrams of each member of the quadrilateral family which includes quadrilaterals, parallelograms, trapezoids, rectangles, rhombuses, squares.

b. A regular polygon is a polygon all of whose sides have the same length and all of whose angles have the same measure.

c. Draw a quadrilateral on your paper. Measure each of the angles and find their sum. Repeat the experiment.

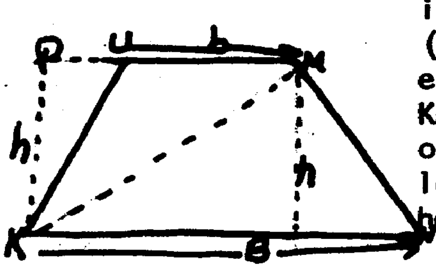
What seems to be true of the sum of the angle measures of a quadrilateral? (360^{o})
Let us try to prove the statement: The sum of the angle measures of a quadrilateral is 360^{o}.
Draw a quadrilateral, ABCD, and its diagonal, AC. Does the diagonal divide the quadrilateral into two triangles? Can any quadrilateral be divided into two triangles? What is the sum of angles measured in each triangle? (180^{o}) What is the sum of the angle measures of the two triangles? (360^{o}) Would the sum of the angle measures of the triangles equal the sum of the angle measures of the quadrilateral? (Yes) Then what is the sum of the angle measures of ABCD? of any quadrilateral? (360^{o})

16. The Trapezoid

a. A trapezoid is a quadrilateral with exactly one pair of opposite sides parallel.

How do you know that a trapezoid has four sides, four vertices, and two diagonals, and that the sum of its angle measures 360^{o}? (Properties of all quadrilaterals.)

b. You know that the area enclosed by any geometric figure can be found if the figure can be divided into non-overlapping triangles, rectangles, or squares.



Does diagonal KM of trapezoid KLMN divide it into two non-overlapping triangles? (Yes) Then is the area of trapezoid KLMN equal to the sum of the areas of KLM and KMN? (Yes) The length of the lower base of the trapezoid is represented by B, the length of the upper base by b, and the height by h.

Examine triangle KMN. What is the length of its base? (B) What is its height? (h) Then what is its area? ($\frac{1}{2}$ Bh) Now examine triangle KLM. What is the length of its base? What is its height? Since the area enclosed by the trapezoid is equal to the sum of the area of the triangles, we can say:

area of a trapezoid = $\frac{1}{2}Bh + \frac{1}{2}bh$

Then what principle allows us to write the following formula for finding the area of a trapezoid? (Distributive)

$A = \frac{1}{2}h(B + b)$

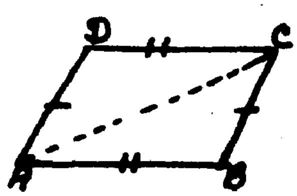
17. The Parallelogram

a. A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

b. Let us try to prove the statement: A diagonal of a parallelogram divides it into two congruent triangles. Draw parallelogram ABCD and construct its diagonal, AC. You already know that if two parallel lines are intersected by a transversal the alternate interior angles are equal in measure. You know that AD∥BC. Are angles BCA and DAC alternate interior angles formed by these parallel lines and the transversal AC? (Yes) Then what is true of m∠BAC and m∠DAC? (They are equal.) Name the parallel lines and the transversal that form alternate interior angles BAC and DCA. Then m∠BAC = m∠DCA? (Yes)
Now, in the two triangles formed by the parallelogram and its diagonal, you have two angles and the included side of one equal in measure to two angles and the included side of the other.
You have also proved the statement: The opposite sides of a parallelogram are equal in length.

You know that two pairs of angles in the triangles have equal measures. Then what must be true of the third pair of angles? (Their measures are equal.) Can you also say that the opposite pairs of angles of a parallelogram are equal? (Yes)

c. Suppose we know that the opposite sides of quadrilateral ABCD have the same length. Is the quadrilateral a parallelogram? (Yes) The only way we can prove that ABCD is a parallelogram is to prove the opposite sides parallel. Is the diagonal AC a transversal? (Yes)



Which two alternate interior angles must have the same measure to prove AB∥DC? (Angles BAC and DCA) Which two angles must have the same measure to prove AD∥BC? (DAC, BCA) Can we conclude that each of these pairs of angles has the same measure if we prove triangle ACD, triangle CAB? (Yes) You know that the opposite sides of ABCD are equal in length. Then what principle can you use to prove the two triangles congruent? (Three sides of one have same measures as three sides of the other.)

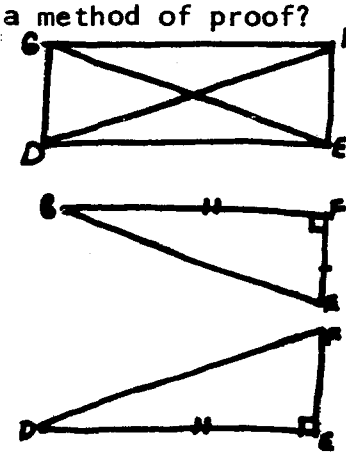
d. In the above exercise, you completed an analysis of a problem, but you did not prove the statement. You did, however, discover what must be done to prove it. Now prove that: If the opposite sides of a quadrilateral have the same measure, it is a parallelogram. The steps of the proof should develop from the fact that a diagonal of the given quadrilateral divides it into the congruent triangles.

18. The Rectangle

a. Since a rectangle is a quadrilateral, it inherits all the properties of a quadrilateral. It also inherits the properties proved for a parallelogram. Name these. (Review definition of rectangle: A rectangle is a parallelogram in which the angles are right angles.)

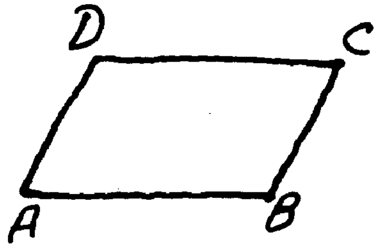
b. Draw a rectangle and measure its diagonals. Do the diagonals have the same length? (Yes) Try the experiment with several rectangles of different shapes. What seems to be true? (Diagonals of a rectangle have same length.)

c. Can we analyze the problem to find a method of proof? At the right is a diagram of a rectangle with its two diagonals. Would DF and EG have the same length if we proved they are corresponding sides of congruent triangles? (Yes) Diagonal EG is a side of triangle GFE, and DF is a side of triangle DEF. At the right, we show the triangles separately. Now let us prove that: The diagonals of a rectangle have the same length. Is mDE=mGF? Why? (Opposite sides of a rectangle have same length.) Angles DEF and GFE are right angles. Then does m$\angle$DEF=m$\angle$GFE? (Yes) Which side of the rectangle is common to both triangles? (FE) State the principle you will use to conclude that triangle DEF$\cong$triangle GFE. (Two sides and included angle of one triangle have same measures as two sides and included angle of the other.)



d. If we prove that the diagonals of a rectangle bisect each other, may we assume that the diagonals of a parallelogram bisect each other. (No) However, if we prove that the statement is true for parallelograms, may we assume that rectangles inherit this property? (Yes) (Emphasize that proving something true for a special type of figure does not make it true for a more general figure.)

e. Look at the diagram of a parallelogram at the left. In triangles ABO and CDO, OD corresponds to OB, and OA to OC. If we prove these triangles congruent, what can we say about the corresponding sides? (Their lengths are equal.) Then we shall have proved: The diagonals of a parallelogram bisect each other. You know that AB∥DC. Is DB a transversal? (Yes) Why do these facts make m∠ABD and m∠CDB equal? (They are alternate interior angles formed by parallel lines and a transversal.) Is AC also a transversal of AB and DC? (Yes) Then which other pair of angles must have equal measures? (∠s ABD and CDB) How do the lengths of AB and DC compare? (They are equal.) State the principle that permits you to conclude that triangle ABO ≅ triangle CDO. Therefore, mOD = mOB, and mOA = mOC. Have you completed the desired proof? (Two angles and included side of one triangle have same measures as two angles and included side of the other.)

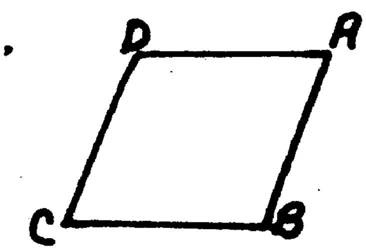


f. Do you think it is true that: A parallelogram with one right angle is a rectangle? What would you have to prove in order to establish this statement? (That all the angles are right angles.) What properties concerning the angles of a parallelogram would be helpful? (Opposite pair of angles have equal measures; consecutive angles are supplementary.)

19. The Rhombus and the Square

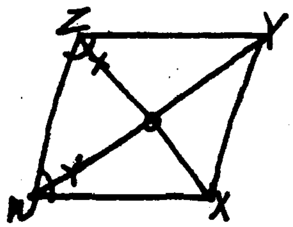
a. A rhombus is a parallelogram all of whose sides have the same length. List the properties that a rhombus inherits from the other quadrilaterals.

b. Draw a rhombus like the one at the right, with each side $1\frac{1}{2}$ inches long and an acute angle equal to 70°. Measure angles DAC and BAC. Then measure angles DCA and BCA. Do you find that the diagonal bisects the opposite angles of the rhombus? (Yes)



The experiment indicates that a diagonal of a rhombus bisects the opposite angles. Now we should like to prove this to be true. Given rhombus ABCD, is it true that all of its sides have the same length? Can you conclude that mAD=mDC? Then is triangle ADC an isosceles triangle? In an isosceles triangle, you know that the angles opposite the sides of equal length have equal measures. What other things do you know about a rhombus?

c. Let us try to prove that the diagonals of a rhombus are perpendicular. Examine the rhombus at the left. If m∠WZY is x and m∠ZWX is y, what can you say about the sum of x and y? (Equals 180°) You know that the diagonals of a rhombus bisect the angles. Then does m∠WZO=$\frac{1}{2}$x and m∠ZWO=$\frac{1}{2}$y? (Yes) What does $\frac{1}{2}$x +$\frac{1}{2}$y equal? 90°. Now examine triangle ZOW. You know that the sum of the measures of∠WZO and∠ZWO equals 90°. Then what is m∠ZOW? (90°) Have you proved that the diagonals of a rhombus are perpendicular? (Yes)



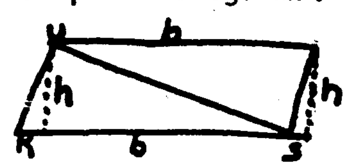
d. A square is a parallelogram all of whose sides have the same length and all of whose angles are right angles.

e. What kinds of triangles are formed by the diagonal of a rhombus? (Isosceles) of a rectangle? (Right) of a parallelogram? (Scalene) of a quadrilateral? (Scalene) In which figures may you assume the triangles are congruent? (All except quadrilateral.)

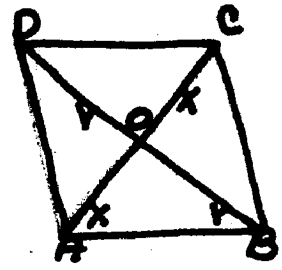
f. Is the definition of a square above a complete definition? (No) Another way to state the definition is: If a parallelogram has all of its sides the same length and all of its angles right angles, it is a square. State the converse. Is the converse true? (Yes) (If a parallelogram has all of its sides the same length, it is a rhombus. If a parallelogram is a rhombus, it has all of its sides the same length.)

20. Area of a Parallelogram and Rhombus

a. Do you know the formula for the area of a parallelogram? Let us derive this formula, using what we already know about areas of triangles and properties of parallelograms. Parallelogram RSTU has a height whose length is h and a base length of b. Are the opposite sides of a parallelogram equal in length? (Yes) Then does mUT=h? (Yes) Does diagonal SU divide the parallelogram into two non-overlapping triangles? (Yes) What is the area of triangle RSU? ($\frac{1}{2}$bh) Is the area of the parallelogram equal to the sum of the areas of the triangles? (Yes) If A is the area of the parallelogram. we may say:



$$A = \tfrac{1}{2}bh + \tfrac{1}{2}bh \quad \text{or} \quad A = bh$$



b. The figure ABCD is a rhombus with its diagonals AC and BD. Do the diagonals bisect each other? (Yes) Then we may indicate the lengths of the segments of the diagonals as in the diagram, making the lengths of the diagonals 2x and 2y.

Let us try to find the area of a rhombus in terms of the lengths of its diagonals. What kind of angle is each of the angles at O? (Right angles) Then what is the area of triangle AOD? ($\frac{1}{2}xy$) What is the area of each of the other triangles formed? What is the area of the rhombus in terms of x and y? Can this be written as $A=2xy$, or $A=\frac{1}{2} \times 2x \times 2y$? (Yes) Would it be correct to say the area of a rhombus equals one-half the product of the lengths of its diagonals?

c. On paper, write all of the quadrilaterals discussed. Under each, list all the properties you have been given or have proved about these figures.

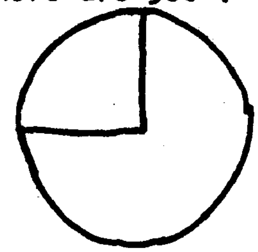
Quadrilateral: 4 sides, 4 vertices, 4 angles, 2 diagonals, sum of angles measure 360°.
Parallelogram: all foregoing, plus: both pairs of opposite sides parallel; opposite sides equal in length; opposite pairs of angles have equal measures; consecutive angles are supplementary; diagonal divides it into two congruent triangles; diagonals bisect each other.
Trapezoid: Properties of the quadrilateral plus one pair of opposite sides parallel.
Rectangle: All properties of a quadrilateral and parallelogram plus all angles are right angles, diagonals have equal lengths.
Rhombus: All properties of a quadrilateral and parallelogram plus all sides have the same length, diagonals bisect opposite angles, diagonals are perpendicular.
Square: All properties of a quadrilateral, parallelogram, rectangle, and rhombus.

21. Circle

a. A circle is a plane figure consisting of the set of all points at a given distance from a point in the same plane.

b. Review the definition of the radius, diameter, arc, and the definition of a semicircle.

c. You know that in a complete revolution there are 360°. The diagram shows a right angle in a circle with its vertex at the center and radii for sides. An angle that has its vertex at the center of the circle is called a central angle. An arc of a circle has the same measure as its central angle.



An arc of a circle has the same measure as its central angle. Another principle we accept is: A central angle has the same measure as its arc.

d. You know that a regular polygon has all of its sides the same length and all of its angles equal in measure. When a figure is inscribed in a circle, all of its vertices are points on the circle and all of its sides lie inside the circle.

e. Draw a large circle on your paper. Draw two diameters that are perpendicular. What is the measure of each central angle? Connect the ends of the diameters with line segments. What is the name of the figure formed? Prove that the figure is a square by proving that certain triangles are congruent.

In any two of the small triangles formed, two sides and included angle of one have same measures as two sides and included angle of the other. Therefore, sides of the inscribed polygon have same length. Acute angles of triangles measure 45°. Pair of adjacent angles at ends of diameters measure 90°. Therefore, angles of polygon are right angles. Figure is a square.)

22. Similar Triangles

a. Two triangles having the same shape, but not necessarily the same size, are called similar triangles.

Two triangles are similar if the measures of corresponding angles are equal and the ratios of the lengths of corresponding sides are equivalent.

b. Draw two right triangles whose acute angles have measures of 30° and 60°. Will the measures of all pairs of corresponding angles be equal? (Yes) Are the ratios of the lengths of corresponding sides equivalent? (Yes) Then what is the relationship between the two right triangles? (They are similar)

(1) You know that a proportion is a statement that two ratios are equivalent. To simplify the statement "The ratios of the lengths of corresponding sides are equivalent," we may say "Corresponding sides are proportional."

(2) If the measures of corresponding angles of two triangles are equal, the triangles are similar.

(3) We use the symbol " " to mean "is similar to."

(4) Two right triangles have a pair of corresponding acute angles equal in measure. Are the measures of the third pair of angles equal? Why? (The sum of the measures of the angles of any triangle equals 180°.) Then, if an acute angle of one right triangle has the same measure as an acute angle of another right triangle, the triangles are similar.

23. Pythagorean Rule

a. The two sides of a right triangle which form the right angle are called the <u>Legs</u> of the right triangle.

The side opposite the right angle is called the <u>hypotenuse</u>.

(1) When a right angle is named "ABC" it is customary to label the vertex of the right angle C. The length of the side opposite each vertex is denoted by the corresponding small letter. (For example, the side opposite $\angle$ would be labeled c.)

(2) Is the hypotenuse always the longest side of a right triangle? (Yes) Why? (It is opposite the right angle which is always the largest angle.)

b. Draw a right triangle with legs 8 and 15 units in length. Is the hypotenuse 17 units in length? Does $8^2 + 15^2 = 17^2$?

The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse. This is the "right triangle rule," or the "Pythagorean rule."

$$a^2 + b^2 = c^2$$

24. Square Root

a. In the formula $a^2 + b^2 = c^2$ we can find c^2 if we know the values of a and b. We also can find a^2 if we know the values of b and c, and b^2 if we know the values of a and c. But in many cases it is not c^2 or b^2 that we desire to find; it is c or a or b that we must find. To do this we must know how to find the value of c when the value of c^2 is known, for instance.

(1) We know that 5^2 means $5 \cdot 5$. We call 25 the "square of 5."

(2) Starting with a number such as 169, we should like to "unsquare" it. That is, we should like to find the number that when multiplied by itself results in 169. Such a number is called a <u>square</u> <u>root</u>.

b. A number such as 25 is called a perfect square, because the value of its square root is a whole number; that is, $25 = 5$. Just as addition and subtraction are inverse operations, so finding the square of a number and finding the square root of a number are inverse operations. The symbol for square root is "$\sqrt{\ }$"; it is read "the square root of."

In this situation, we are speaking of the positive square root only.

25. Approximating Square Roots

a. Draw a square with a length of 1 inch. To find the length of its diagonal, we would have to know the value of a and b, which we do know. In the formula, replace a with 1 and b with 1.

$$1^2 + 1^2 = c^2$$
$$2 = c^2$$
$$\sqrt{2} = c$$

(1) Between what two perfect squares does 2 lie? 1 and 4. Then $\sqrt{2}$ is greater than 1 and less than 2. Can we write this relationship as follows? (Yes)

$$1 < \sqrt{2} < 2$$

(2) Do you think that $\sqrt{2}$ is closer to 1 or to 2? Try 1.4 as the square root. What does $(1.4)^2$ equal? Is $\sqrt{2}$ greater than 1.4? Try 1.5. What does its square equal? Is $\sqrt{2}$ less than 1.5? Then we can write:

$$1.4 < \sqrt{2} < 1.5$$

(3) From these results, would you say that $\sqrt{2}$ lies very close to 1.4, midway between 1.4 and 1.5, or close to 1.5? Try 1.41. Square it. Try 1.42 and square it.

(4) Would you say that $\sqrt{2}$ lies close to 1.41, about midway between 1.41 and 1.42, or close to 1.42? What is the square of 1.415? 1.414? Write an inequality that orders $\sqrt{2}$ between two numbers each with three decimal places.

(5) Carry the work on one step farther so that you find $\sqrt{2}$ between two 4-place decimal numbers. A number such as $\sqrt{2}$ is like the number, π. No matter how many decimal places you carry the work out to, the decimal expression will not become repeating, nor will it end, or terminate.

b. To illustrate another method we will find $\sqrt{39}$ correct to the nearest thousandth. Between what two perfect squares does 39 lie? (36 and 49) Is $\sqrt{39}$ closer to 6, or 7? Try 6.3 as an approximation to one of the two equal factors of 39. We use division to find the other factor and carry the work to one more decimal place than there is in the divisor. Since the quotient, 6.19, is smaller than the divisor, you know that 6.3 was too large a choice.

Do you think the correct square root is one-half of the way between 6.19 and 6.3? What is the difference? one-half the difference? Add this to the smaller factor, either the divisor or the quotient. Then you have 6.19 + .055 as one factor. Use the sum as a new guess for the square root and divide again. Again find the difference between the divisor and the quotient and add one-half of it to the smaller factor.

c. A third method of finding approximations to square roots is by the use of a table of squares and square roots.

IV. ACTIVITIES:

Because of the wide variety and number of activities in the unit on geometry, they are incorporated into the subject matter outline. This is desirable because in developing the various concepts in this unit three or four activities may be needed to properly demonstrate the concept under discussion.

APPENDIX G

DATA SHOWING RAW SCORES AND DIFFERENCE BETWEEN TOTAL RAW SCORES ON ARITHMETIC REASONING (A), ARITHMETIC CONCEPTS (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES FOR QUARTILE I-- SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

EXPERIMENTAL GROUP									CONTROL GROUP									
September, 1962				April, 1963					September, 1962				April, 1963					
A	B	C	Total	A	B	C	Total	Dif. in Totals	A	B	C	Total	A	B	C	Total	Dif. in Totals
8	10	9	27	17	13	9	39	12	9	9	8	26	17	17	20	54	28
9	10	10	29	12	10	17	39	10	9	9	9	27	11	22	4	37	10
10	6	14	30	14	9	8	31	1	8	6	17	31	26	20	28	74	43
7	11	14	32	19	17	20	56	24	12	12	8	32	14	11	16	41	9
14	12	7	33	13	16	19	48	15	12	8	12	32	10	11	11	32	0
9	5	20	34	19	10	12	41	7	15	6	12	33	19	14	20	53	20
12	8	14	34	20	13	15	48	14	6	16	13	35	15	18	27	60	25
9	13	12	23	13	11	17	41	7	10	11	14	35	14	17	18	49	14
14	7	14	35	15	14	11	40	5	10	15	10	35	12	10	20	42	7
10	15	10	35	26	13	23	62	27	14	10	12	36	14	11	13	38	2
12	9	14	35	27	18	30	75	40	11	10	16	37	19	16	24	59	22
9	8	19	36	14	12	15	41	5	14	6	17	37	20	13	38	71	34
9	10	17	36	16	17	27	73	37	15	11	11	37	20	12	21	53	16
14	10	13	37	31	20	28	79	42	16	12	9	37	31	23	33	87	50
8	7	23	38	20	16	17	53	15	9	11	18	38	12	18	17	47	9
10	15	13	38	16	12	13	41	3	15	14	9	38	15	10	11	36	-2
10	6	22	38	18	9	26	53	15	11	16	11	38	15	19	26	60	22
10	10	18	38	7	8	19	34	-4	15	11	13	39	20	12	25	57	18
13	5	20	38	13	9	17	39	1	15	10	14	39	13	12	19	44	5
11	9	18	38	12	18	26	56	18	9	15	16	40	23	17	29	69	29
8	15	17	40	16	15	19	50	10	14	14	12	40	13	17	19	49	9
8	11	22	41	12	14	21	47	6	20	8	12	40	13	12	15	40	0
11	10	20	41	20	12	12	44	3	21	6	13	40	23	18	25	66	26
17	11	13	41	16	15	17	48	7	10	10	20	40	21	22	29	72	32

APPENDIX G (Continued)

EXPERIMENTAL GROUP September, 1962				April, 1963					CONTROL GROUP September, 1962				April, 1963				
A	B	C	Total	A	B	C	Total	Dif. in Totals	A	B	C	Total	A	B	C	Total	Dif. in Totals
21	8	13	42	13	2	8	23	-19	12	13	18	43	19	22	39	80	37
16	9	17	42	18	12	13	43	1	12	13	18	43	16	15	15	46	3
13	11	18	42	13	14	22	49	7	11	7	25	43	22	21	25	68	25
13	10	19	42	19	17	29	65	23	13	11	20	44	18	20	8	46	2
16	10	17	43	19	13	16	48	5	16	5	23	44	12	9	22	43	-1
11	10	22	43	16	16	28	60	17	14	15	16	45	21	24	20	65	20
15	12	17	44	22	13	16	51	7	18	12	16	46	21	12	26	59	13
15	12	17	44	22	13	16	51	7	11	15	20	46	13	23	38	74	28
23	14	7	44	14	19	11	44	0	18	12	16	46	14	17	13	44	-2
17	11	16	44	27	15	21	63	19	15	15	16	46	14	17	16	47	1
15	17	12	44	13	14	12	39	-5	13	7	26	46	16	10	26	52	6
16	14	14	44	20	16	27	63	19	12	12	22	46	15	19	20	54	8
12	10	23	45	18	8	16	42	-3	17	6	23	46	10	12	21	43	-3
14	13	18	45	21	17	23	61	16	12	12	22	46	9	13	19	41	-5
15	12	18	45	18	16	22	56	11	10	19	18	47	30	23	40	93	46
17	14	14	45	19	15	23	57	12	15	14	18	47	15	13	10	38	-9
15	11	19	45	11	10	21	42	-3	14	13	20	47	27	20	35	82	35
22	9	14	45	18	18	17	53	8	18	21	19	48	26	23	25	74	26
18	12	15	45	17	18	15	50	5	26	7	15	48	20	15	21	56	8
19	10	16	45	25	18	19	62	17	15	15	18	48	18	14	14	46	-2
15	16	14	45	20	19	23	62	17	16	17	16	49	21	21	38	80	31
14	11	21	46	19	12	12	43	-3	15	11	23	49	19	12	34	65	16
16	9	21	46	17	15	34	66	20	16	12	21	49	29	21	24	74	25
15	14	17	46	15	19	28	62	16	26	14	9	49	22	18	14	54	5
19	11	17	47	16	14	27	57	10	16	16	17	49	21	22	28	71	22
19	10	18	47	10	12	19	41	-6	13	10	26	49	21	12	18	51	2
20	9	18	47	18	11	19	48	1	21	16	12	49	21	17	9	47	-2
14	9	24	47	17	13	19	49	2	20	14	15	49	23	18	23	64	15

APPENDIX G (Continued)

EXPERIMENTAL GROUP September, 1962 A	B	C	Total	April, 1963 A	B	C	Total	Dif. in Totals	CONTROL GROUP September, 1962 A	B	C	Total	April, 1963 A	B	C	Total	Dif. in Totals
19	12	16	47	15	10	11	36	-11	14	11	24	49	16	15	26	57	8
15	12	20	47	26	19	31	76	29	9	14	26	49	21	16	37	74	25
15	12	20	47	26	19	31	76	29	17	15	18	50	16	17	22	55	5
18	10	20	48	14	21	17	52	4	8	10	32	50	24	22	40	86	36
17	14	17	48	19	21	28	68	20	21	10	19	50	30	17	40	87	37
15	8	25	48	19	24	36	79	31	21	12	17	50	18	22	23	63	13
17	10	21	48	27	18	22	67	19	24	7	19	50	23	16	22	61	11
16	13	19	48	17	12	18	47	-1	18	13	19	50	20	15	20	55	5
19	13	16	48	18	11	23	52	4	16	17	17	50	27	16	26	69	19
24	15	9	48	19	19	23	61	13	20	12	18	50	18	15	21	54	4
21	9	19	49	25	20	18	63	14	13	12	26	51	21	23	34	78	27
18	11	20	49	16	9	29	54	5	14	14	23	51	20	15	25	60	9
18	8	23	49	22	14	35	71	22	19	16	16	51	26	17	26	69	18
17	13	19	49	22	13	25	60	11	13	9	29	51	13	12	30	55	4
12	10	27	49	23	14	24	61	12	21	13	17	51	18	13	15	46	-5
14	16	19	49	19	22	13	54	5	25	17	9	51	26	21	37	84	33
12	14	23	49	20	11	30	61	12	13	15	23	51	5	17	25	47	-4
14	9	27	50	11	20	30	61	11									
15	12	23	50	14	15	32	61	11									
16	10	24	50	31	18	28	77	27									
17	20	23	50	24	22	28	74	24									
17	11	22	50	21	11	28	60	10									
20	9	22	51	23	16	28	67	16									
19	10	22	51	16	14	24	54	3									
19	10	22	51	27	24	25	76	25									
22	15	14	51	26	17	19	62	11									
15	11	25	51	19	15	37	71	20									
24	12	15	51	21	16	25	62	11									

APPENDIX H

DATA SHOWING RAW SCORES AND DIFFERENCE BETWEEN TOTAL RAW SCORES ON ARITHMETIC REASONING (A), ARITHMETIC CONCEPTS (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES FOR QUARTILE II-- SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

EXPERIMENTAL GROUP									CONTROL GROUP								
September, 1962				April, 1963				Dif. in Totals	September, 1962				April, 1963				Dif. in Totals
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
18	15	19	52	18	17	20	55	3	27	16	9	52	24	17	34	75	23
19	13	20	52	12	14	19	45	-7	14	14	24	52	23	18	28	69	17
21	10	21	52	16	17	37	70	18	26	10	16	52	23	20	19	62	10
15	12	25	52	14	14	33	61	9	15	23	14	52	28	24	34	86	34
16	14	22	52	19	18	34	71	19	18	12	22	52	15	12	26	53	1
19	13	20	52	13	24	37	74	22	18	7	27	52	20	17	32	69	17
13	11	28	52	11	16	28	55	3	16	16	20	52	22	25	41	88	36
18	16	19	53	16	14	21	51	-2	22	16	15	53	33	19	21	73	20
24	14	15	53	28	22	30	80	27	21	12	20	53	18	22	16	56	3
17	11	26	54	16	15	34	65	11	16	13	24	53	24	17	31	72	19
19	16	19	54	20	16	12	48	-6	23	9	21	53	30	21	27	78	25
19	14	21	54	16	17	30	63	9	21	15	18	54	24	15	34	73	19
17	17	20	54	24	15	33	72	18	12	14	28	54	17	15	32	64	10
13	10	32	55	27	21	31	79	24	22	11	21	54	23	19	32	74	20
18	17	20	55	34	24	24	82	27	17	8	29	54	23	19	33	75	21
17	13	25	55	24	21	35	80	25	27	14	14	55	24	22	38	84	29
13	17	25	55	28	19	23	70	15	16	19	20	55	17	18	35	70	15
22	17	16	55	22	23	19	64	9	16	14	23	55	16	12	33	61	6
14	15	26	55	18	21	34	73	18	17	15	23	55	20	15	15	50	-5
17	12	26	55	20	18	15	53	-2	20	13	22	55	20	17	24	61	6
24	15	16	55	13	17	24	54	-1	20	14	21	55	24	14	29	67	12
16	14	25	55	22	29	40	91	36	20	13	23	56	27	21	28	76	20
14	13	29	56	17	10	21	48	-8	15	16	25	56	16	15	35	66	10
18	12	26	56	12	16	32	60	4	18	13	25	56	22	19	26	67	11
17	14	25	56	26	23	35	84	28	20	9	27	56	16	16	20	52	-4

APPENDIX H (Continued)

EXPERIMENTAL GROUP									CONTROL GROUP									
September, 1962				April, 1963				Dif. in Totals	September, 1962				April, 1963				Dif. in Totals
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
19	10	27	56	15	15	24	54	-2	22	14	20	56	25	15	38	78	22
15	13	28	56	13	13	28	54	-2	16	16	24	56	28	24	45	97	41
28	10	18	56	20	15	25	60	4	19	14	23	56	18	15	24	57	1
21	16	20	56	24	22	37	83	27	19	9	28	56	19	16	30	65	9
17	17	23	57	23	18	25	66	9	12	21	23	56	29	25	30	84	28
15	15	27	57	14	14	31	59	2	24	12	21	57	27	15	38	80	23
19	13	25	57	15	13	19	47	-10	18	14	25	57	18	14	38	70	13
14	15	28	57	32	19	32	83	26	25	14	18	57	22	16	13	51	-6
22	16	19	57	19	15	31	65	8	18	10	29	57	25	13	30	68	11
18	14	25	57	25	23	36	84	27	18	19	20	57	13	12	16	41	-16
20	10	28	58	21	17	30	68	10	22	17	18	57	30	27	37	94	37
10	14	35	58	26	18	32	76	18	23	12	23	58	21	12	21	54	-4
19	13	26	58	24	22	34	80	22	20	14	24	58	25	17	29	71	13
19	15	24	58	21	23	37	81	23	25	19	14	58	25	25	33	83	25
17	15	26	58	21	13	43	87	29	23	13	23	59	35	19	23	77	18
33	11	15	59	21	12	20	53	-6	17	21	21	59	12	18	32	62	3
19	14	26	59	25	16	19	60	1	28	14	17	59	23	16	33	72	13
18	16	25	59	27	20	30	77	18	26	19	14	59	32	25	39	96	37
13	11	35	59	22	17	36	75	16	25	11	23	59	38	18	32	78	19
14	18	27	59	23	19	19	61	2	22	12	25	59	29	16	31	76	17
23	8	28	59	17	17	24	58	-1	17	18	25	60	15	25	29	69	9
20	17	22	59	23	24	23	70	11	17	16	27	60	30	26	31	87	27
26	13	20	59	23	24	25	72	13	22	15	23	60	23	16	34	73	13
18	11	30	59	20	17	35	72	13	21	20	19	60	25	16	25	66	6
22	15	23	60	21	19	27	67	7	25	18	17	60	24	19	33	76	16
18	17	25	60	29	25	34	88	28	21	18	21	60	20	23	40	83	23
0	22	38	60	29	22	32	83	23	23	15	22	60	27	21	28	76	16
15	17	28	60	28	23	33	84	24	15	18	28	61	26	24	31	81	20
21	17	23	61	22	18	27	67	6	22	18	21	61	19	26	35	80	19

APPENDIX H (Continued)

EXPERIMENTAL GROUP									CONTROL GROUP								
September, 1962				April, 1963				Dif. in Totals	September, 1962				April, 1963				Dif. in Totals
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
20	15	26	61	21	11	25	57	-4	27	11	23	61	24	18	33	75	14
24	13	24	61	16	12	32	60	-1	20	20	21	61	23	17	21	61	0
20	12	29	61	24	23	41	88	27	19	18	25	62	28	23	32	83	21
20	16	26	62	17	14	36	67	5	16	22	24	62	25	23	21	69	7
19	15	28	62	23	19	36	78	16	14	15	33	62	28	19	40	87	25
25	19	18	62	23	24	29	76	14	12	19	31	62	43	19	31	93	31
24	17	21	62	22	18	25	65	3	18	19	25	62	21	20	29	70	8
20	21	21	62	33	23	31	87	25	15	15	32	62	19	16	35	70	8
26	13	23	62	22	16	23	61	-1	18	16	28	62	23	14	41	78	16
17	13	31	62	20	19	32	71	9	14	6	42	62	20	13	27	60	-2
23	12	27	62	23	23	36	82	20	24	13	25	62	26	25	32	83	21
17	15	30	63	13	19	36	68	5	21	16	25	62	31	22	35	88	26
22	22	19	63	30	19	23	72	9	19	12	31	62	21	22	41	84	22
19	14	31	64	31	23	40	94	30	22	19	22	63	21	19	22	62	-1
24	14	26	64	21	16	13	50	-14	16	18	29	63	14	23	31	68	5
20	16	28	64	22	20	26	68	4	19	13	31	63	36	27	33	96	33
16	21	27	64	22	19	23	64	0	23	14	26	63	29	26	37	92	29
22	20	22	64	27	17	26	70	6	23	14	27	64	23	17	28	68	4
20	17	27	64	22	22	36	80	16	20	17	27	64	27	16	39	82	18
26	17	21	64	26	22	37	85	21	18	15	31	64	16	12	36	64	0
									20	14	30	64	26	24	38	88	24
									20	19	25	64	25	23	43	91	27

APPENDIX I

DATA SHOWING RAW SCORES AND DIFFERENCE BETWEEN TOTAL RAW SCORES ON ARITHMETIC REASONING (A), ARITHMETIC CONCEPTS (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES FOR QUARTILE III--SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

EXPERIMENTAL GROUP								Dif. in Totals	CONTROL GROUP								Dif. in Totals
September, 1962				April, 1963					September, 1962				April, 1963				
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
17	13	35	65	33	22	43	98	33	28	17	20	65	30	19	19	68	3
23	16	26	65	23	16	22	61	-4	18	23	24	65	31	26	32	89	24
17	17	31	65	18	15	22	55	-10	24	13	28	65	23	19	35	77	12
23	16	26	65	23	19	35	77	12	21	14	30	65	23	22	27	72	7
26	16	23	65	27	23	23	73	8	30	17	18	65	31	21	26	78	13
22	18	25	65	35	16	39	90	25	29	19	18	66	36	24	34	94	28
17	16	33	66	27	23	33	83	17	24	12	30	66	24	16	34	74	8
22	21	23	66	26	23	26	75	9	19	20	27	66	17	23	34	74	8
21	19	26	66	24	20	30	74	8	19	22	25	66	34	18	38	90	24
28	18	20	66	11	20	26	57	-9	25	15	26	66	33	26	31	90	24
19	12	36	67	26	22	25	73	6	20	20	26	66	15	19	23	57	-9
25	14	28	67	18	14	27	59	-8	20	17	29	66	33	17	36	86	20
16	21	30	67	30	27	39	96	29	29	16	21	66	29	23	24	76	10
13	21	33	67	17	21	35	73	6	17	18	31	66	27	18	27	72	6
21	13	33	67	24	15	23	62	-5	25	14	27	66	30	20	36	86	20
20	14	34	68	17	14	11	42	-26	25	17	34	66	20	13	30	63	-3
22	15	31	68	26	20	29	75	7	23	14	30	67	35	25	40	100	33
26	15	27	68	22	16	31	69	1	23	14	30	67	26	13	30	79	12
28	15	26	69	15	14	20	49	-20	25	21	21	67	23	16	26	65	-2
29	13	27	69	20	13	14	47	-22	27	13	27	67	25	17	35	77	10
27	15	27	69	19	15	28	62	-7	21	13	33	67	14	16	30	60	-7
21	19	29	69	25	18	26	69	0	24	18	25	67	14	20	10	44	-23
25	16	28	69	23	25	41	89	20	22	13	32	67	16	19	33	68	1
30	18	21	69	36	26	43	105	36	15	16	36	67	33	24	47	104	37
26	13	31	70	27	18	38	83	13	24	13	31	68	25	18	31	74	6

APPENDIX I (Continued)

EXPERIMENTAL GROUP September, 1962 A	B	C	Total	April, 1963 A	B	C	Total	Dif. in Totals	CONTROL GROUP September, 1962 A	B	C	Total	April, 1963 A	B	C	Total	Dif. in Totals
24	19	27	70	21	21	21	63	-7	27	14	27	68	27	22	41	90	22
25	20	25	70	20	21	22	63	-7	24	16	28	68	34	21	39	94	26
25	14	32	71	28	18	39	85	14	24	13	32	69	35	20	38	93	24
22	17	32	71	27	19	32	78	7	20	21	28	69	29	22	34	85	16
25	21	25	71	24	16	24	64	-7	18	12	39	69	22	17	34	73	4
21	21	29	71	24	29	23	76	5	22	16	31	69	25	18	32	75	6
24	19	28	71	38	19	38	95	24	26	14	30	70	20	21	30	71	1
25	17	29	71	28	19	36	83	12	24	14	32	70	27	17	32	76	6
26	24	31	71	36	30	45	111	40	29	13	28	70	34	21	43	98	28
25	22	25	72	17	24	26	67	-5	26	19	26	71	24	19	39	82	11
24	22	26	72	21	15	28	64	-8	23	20	28	71	45	31	40	116	45
22	19	31	72	36	24	34	94	22	31	14	27	72	32	18	34	84	12
21	16	35	72	27	24	40	91	19	26	23	23	72	36	28	23	89	17
30	16	27	73	23	21	30	74	1	33	12	27	72	32	19	31	82	10
26	18	29	73	17	11	22	50	-23	27	20	25	72	30	24	43	97	25
25	15	33	73	29	20	35	84	11	21	18	34	73	27	24	41	92	19
20	19	34	73	22	21	34	77	4	27	19	27	73	41	30	40	111	38
24	17	32	73	18	17	35	70	-3	21	13	39	73	25	22	41	88	15
23	12	38	73	31	18	42	91	18	26	18	29	73	27	23	39	89	16
23	17	33	73	33	21	40	94	21	30	19	24	73	34	21	46	101	28
30	21	23	74	35	24	34	93	19	28	21	25	74	33	24	39	96	22
24	20	30	74	14	23	33	70	-4	25	25	24	74	36	25	34	95	21
25	20	29	74	25	24	33	82	8	23	18	33	74	35	24	41	100	26
34	13	27	74	31	21	29	81	7	33	19	22	74	39	21	40	100	26
25	17	32	74	25	26	33	84	10	28	12	35	75	25	18	36	79	4
17	18	39	74	33	16	32	81	7	22	19	34	75	26	22	44	92	17
23	15	36	74	26	16	41	83	9	25	15	35	75	18	21	36	75	0
26	18	30	74	26	24	32	82	8	25	23	27	75	21	25	35	81	6

APPENDIX I (Continued)

EXPERIMENTAL GROUP								Dif. in Totals	CONTROL GROUP								Dif. in Totals
September, 1962				April, 1963					September, 1962				April, 1963				
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
27	21	26	74	37	23	38	98	24	24	20	31	75	30	25	44	99	24
35	22	18	75	28	22	28	78	3	31	17	27	75	29	23	39	91	16
31	14	30	75	28	23	35	86	11	21	17	37	75	35	25	45	105	30
21	16	38	75	36	23	39	98	23	25	21	29	75	45	33	46	124	49
28	17	30	75	36	20	33	89	14	27	16	33	76	33	19	39	91	15
27	14	34	75	17	14	37	68	-7	25	19	32	76	27	21	32	80	4
21	17	37	75	34	19	38	91	16	21	21	34	76	24	24	45	93	17
21	17	37	75	34	19	38	91	16	29	18	29	76	32	27	39	98	22
22	17	36	75	22	21	39	82	7	25	15	36	76	26	18	26	70	-6
26	19	30	75	29	26	40	95	20	34	21	21	76	22	27	42	91	15
34	15	26	75	31	19	40	90	15	28	13	35	76	34	27	35	96	20
24	23	29	76	43	21	24	88	12	23	19	34	76	37	26	42	105	29
31	16	29	76	22	20	31	73	-3	26	17	34	77	31	24	43	98	21
23	14	39	76	22	19	40	81	5	31	18	28	77	22	25	24	71	-6
27	21	29	77	37	26	37	103	26	21	19	37	77	32	22	45	99	22
22	22	33	77	16	25	28	69	-8	18	24	35	77	36	22	35	93	16
28	14	35	77	20	22	39	81	4	30	20	27	77	29	19	18	66	-11
26	17	34	77	29	23	41	93	16	28	18	32	78	35	26	33	94	16
25	20	32	77	32	23	38	93	16	28	17	33	78	23	20	33	76	-2
28	18	32	78	18	20	33	71	-7	31	19	28	78	30	25	32	87	9
38	19	21	78	28	21	20	69	-9	30	17	31	78	35	28	46	109	31
18	25	35	78	25	23	26	74	-4	25	17	36	78	35	28	41	104	26
25	24	29	78	35	26	39	100	22									
28	19	31	78	28	22	47	97	19									

APPENDIX J

DATA SHOWING RAW SCORES AND DIFFERENCE BETWEEN TOTAL RAW SCORES ON ARITHMETIC REASONING (A), ARITHMETIC CONCEPTS (B), ARITHMETIC COMPUTATION (C), AND TOTAL RAW SCORES FOR QUARTILE IV-- SRA ACHIEVEMENT TEST IN ARITHMETIC--FORM A, SEPTEMBER, 1962 AND FORM B, APRIL, 1963

EXPERIMENTAL GROUP								Dif. in Totals	CONTROL GROUP								Dif. in Totals
September, 1962				April, 1963					September, 1962				April, 1963				
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
31	15	33	79	36	22	32	90	11	25	24	30	79	38	27	31	96	17
24	21	34	79	31	25	40	96	17	35	15	29	79	29	15	27	71	-8
37	13	29	79	38	19	30	87	8	30	18	32	80	35	28	36	99	19
23	22	34	79	31	23	44	98	19	30	22	38	80	30	22	38	90	10
30	14	35	79	24	16	35	75	-4	27	21	32	80	33	20	33	86	6
28	19	32	79	32	26	44	102	23	28	21	31	80	37	25	45	107	27
29	19	31	79	21	21	33	75	-4	25	21	34	80	32	27	44	103	23
29	15	36	80	35	26	37	98	18	28	19	33	80	37	26	40	103	23
36	11	33	80	32	24	36	92	12	30	19	31	80	28	25	47	100	20
36	11	33	80	32	24	36	92	12	32	15	33	80	43	27	44	114	34
22	27	31	80	31	29	29	89	9	26	18	37	81	30	25	44	99	18
27	24	29	80	34	22	36	92	12	30	22	29	81	34	26	40	100	19
31	17	33	81	34	25	38	97	16	31	16	34	81	21	16	40	77	-4
26	16	39	81	27	20	40	87	6	25	24	33	82	29	22	39	90	8
30	16	35	81	37	23	37	97	16	28	19	35	82	29	21	42	92	10
27	21	34	82	36	28	39	103	21	30	19	33	82	33	26	37	96	14
25	19	38	82	28	24	33	85	3	31	13	38	82	31	23	37	91	9
24	25	33	82	10	10	6	26	-56	24	17	41	82	37	28	45	110	28
28	25	29	82	42	25	34	101	19	23	24	35	82	27	27	45	99	17
32	17	33	82	31	17	34	82	0	32	23	27	82	34	26	36	96	14
31	17	34	82	29	24	27	80	-2	28	21	33	82	21	28	43	92	10
32	18	32	82	35	22	44	101	19	28	22	32	82	31	29	40	100	18
27	20	36	83	43	21	37	101	18	28	18	37	83	30	21	32	83	0
28	20	35	83	35	24	41	100	17	26	21	36	83	28	19	43	90	7
27	27	29	83	35	25	38	98	15	24	19	40	83	24	19	31	74	-9

APPENDIX J (Continued)

EXPERIMENTAL GROUP									CONTROL GROUP									
September, 1962				April, 1963				Dif. in Totals	September, 1962				April, 1963				Dif. in Totals
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
24	23	36	83	21	24	43	88	5	26	19	39	84	38	25	38	101	17
30	23	30	83	26	23	41	90	7	27	19	38	84	21	13	35	69	-15
26	22	35	83	31	28	41	100	17	29	17	38	84	32	22	40	94	10
26	20	38	84	43	26	45	114	30	30	18	36	84	36	19	44	99	15
27	19	38	84	17	13	34	64	-20	33	21	30	84	42	26	44	112	28
23	26	35	84	46	31	47	124	40	33	20	31	84	37	26	43	106	22
27	21	37	85	29	25	48	102	17	31	16	37	84	26	23	41	90	6
30	21	34	85	41	26	42	107	22	35	18	32	85	29	15	29	73	-12
31	22	32	85	33	21	40	94	9	30	21	35	86	39	24	45	108	22
26	22	37	85	24	24	43	91	6	32	23	31	86	28	26	37	91	5
29	23	34	86	28	29	37	94	8	31	20	35	86	33	19	34	86	0
27	20	39	86	35	25	43	103	17	29	22	35	86	38	28	40	106	20
35	20	31	86	39	21	42	102	16	39	24	24	87	26	27	18	71	-16
36	16	35	87	28	23	33	84	-3	32	17	38	87	16	14	36	66	-21
30	21	36	87	42	24	40	106	19	28	23	37	88	38	23	49	110	22
27	21	40	88	27	25	43	95	7	30	20	38	88	32	23	43	98	10
35	20	33	88	31	23	45	99	11	31	19	40	90	28	22	43	93	3
30	22	36	88	28	22	40	90	2	37	24	29	90	40	28	41	109	19
26	27	36	89	33	23	49	105	16	36	19	35	90	41	23	39	103	13
29	21	39	89	40	26	40	106	17	35	24	32	91	43	30	34	107	16
31	17	41	89	31	22	40	93	4	38	20	33	91	35	23	40	98	7
31	20	38	89	38	23	40	101	12	30	29	32	91	31	27	44	102	11
30	21	40	91	27	22	42	91	0	32	19	40	91	39	23	40	102	11
26	25	42	93	32	27	45	104	11	25	26	40	91	44	31	46	121	30
30	23	41	94	32	23	35	90	-4	29	18	45	92	40	27	41	108	16
37	23	34	94	40	28	38	106	12	26	29	37	92	36	23	36	95	3
27	28	40	95	33	26	46	105	10	37	25	30	92	43	30	47	120	28
31	20	44	95	37	30	49	116	21	33	20	40	93	41	30	45	119	26
34	22	39	95	30	31	34	85	-10	35	16	42	93	38	26	43	107	14

APPENDIX J (Continued)

EXPERIMENTAL GROUP									CONTROL GROUP								
September, 1962				April, 1963				Dif. in Totals	September, 1962				April, 1963				Dif. in Totals
A	B	C	Total	A	B	C	Total		A	B	C	Total	A	B	C	Total	
29	24	43	96	46	27	44	117	21	30	22	42	94	32	28	40	100	6
32	26	38	96	28	29	42	99	3	31	20	43	94	37	27	44	108	14
35	22	40	97	40	26	42	108	11	37	17	40	94	33	31	47	111	17
32	22	44	98	40	31	48	119	21	31	25	38	94	40	27	41	108	14
33	26	40	99	37	30	45	112	13	35	22	38	95	46	31	46	123	28
32	23	44	99	45	28	49	122	23	29	25	41	95	17	17	35	69	-26
39	23	38	100	32	29	40	101	1	38	22	35	95	40	25	48	113	18
33	28	40	101	44	29	42	115	14	37	20	39	96	36	26	48	110	14
28	32	41	101	38	33	46	117	16	34	24	39	97	40	27	41	108	11
37	24	40	101	35	23	48	106	5	34	24	39	97	44	28	43	115	18
30	29	43	102	36	27	47	110	8	41	22	35	98	42	26	40	108	10
32	29	41	102	43	29	46	118	16	37	21	40	98	47	27	32	106	8
38	24	40	102	40	23	41	104	2	29	28	41	98	43	32	45	120	22
41	28	36	105	40	30	32	102	-3	39	24	37	100	35	25	42	102	2
33	25	47	105	33	30	43	106	1	34	22	44	100	30	24	36	90	-10
34	29	43	106	48	30	45	123	17	40	20	41	101	35	29	47	111	10
41	24	43	108	44	28	50	122	14	32	28	41	101	36	29	46	111	10
41	27	41	109	46	30	41	117	8	37	23	42	102	40	24	25	89	-13
37	36	39	112	38	31	42	111	-1	33	27	42	102	42	31	45	118	16
43	30	44	117	48	34	48	130	13	47	28	50	102	47	28	50	125	23
									27	31	45	103	43	32	46	121	18
									41	25	40	106	39	33	47	119	13
									38	26	42	106	41	31	46	118	12
									40	28	39	107	47	32	45	124	17
									38	28	41	107	42	29	45	116	9
									33	32	43	108	30	30	44	104	-4
									38	27	43	108	49	31	49	129	21
									40	30	41	111	47	32	48	127	16
									38	34	40	112	47	30	47	124	12
									39	30	44	113	48	32	49	129	16
									48	33	44	125	47	34	49	130	5

Box 543
Univ. of Southwestern La.
Lafayette, Louisiana
April 30, 1962

Dear

I am proposing a study in mathematics for the 1962-63 school year with the eighth grade teachers and students of the white public schools of St. Landry Parish, Louisiana. I am sending you this letter and the attached questionnaire that you may decide whether or not you would like to be a participant along with the other eighth grade teachers.

Briefly, the study will be as follows:

Teachers will be placed in two groups--one designated Group A, the Experimental Group and the other Group B, the Control Group. The students who would ordinarily be assigned to you will remain in the same group as you. Teachers in Group A will meet at the Paul Pavy School in Opelousas, Louisiana on the first Monday of each month throughout the school year. The meetings will be from 4:30 p.m. until 5:30 p.m. At these meetings the teachers will be given lesson plans containing selected material from the "new" mathematics program. The content in the lesson plans will be discussed and how to incorporate this material with the textual material will be shown. Teachers in Group B will meet at the same time and place but on the second Monday of each month. These meetings will be devoted to discussions on how to develop meaning and understanding of the material from the ordinary class textbook.

An organizational meeting for both groups will be held on Friday, August 31, 1962 at the Market Street Elementary School, Opelousas, Louisiana at 3:00 p.m. At this meeting the complete program and responsibilities of teachers will be discussed and questions you may have will be answered.

Please return the questionnaire to me assoon as possible so that I may be able to place you in one of the two groups.

Your cooperation and participation in this program is greatly appreciated. I am looking forward to working with you during this coming school year.

Sincerely,

D. G. Joseph

FACT SHEET ON ALL TEACHERS

PARTICIPATING IN THE STUDY

Name of teacher ______________________ School __________________ Age____

Type of degree held____________ Institution granting degree____________

Date degree granted____________ Sex______

Years of teaching experience___________

Years of teaching eighth grade mathematics_________

Other types of teaching experiences____________________________________

Recent in-service work in teaching mathematics (where?, what kind?)

____

Do you teach in a self-contained classroom?_________

Do you teach in a departmentalized situation?__________

Describe____

____

If you are <u>not</u> an upper-elementary major tell what you are certified to teach and whether or not you are working toward certification in the upper-elementary grade.____

____

VITA

Dorris George Joseph was born May 9, 1922 at Palmetto, Louisiana.

He attended school at Palmetto and Melville, Louisiana, graduating from Melville High School in 1939. In 1951, he received the Bachelor of Arts degree from the University of Southwestern Louisiana, Lafayette, Louisiana. Two years later, he was awarded the Master of Education degree from Louisiana State University.

His experience includes two years of teaching at the seventh grade level in Palmetto, Louisiana, ten years supervising student teaching at the Hamilton Laboratory School, University of Southwestern Louisiana, and three years directing the Veteran's Memorial Trade School, Grand Coteau, Louisiana.

He is married to the former Myrtle Spears, and they have three children, Dennis, Donna, and Jeri Lynn.

EXAMINATION AND THESIS REPORT

Candidate: Dorris George Joseph

Major Field: Education

Title of Thesis: The Effects of a Specially Planned Mathematics Program on Pupil Achievement in Eighth Grade Mathematics

Approved:

Thomas R. Landry
Major Professor and Chairman

Max Goodrich
Dean of the Graduate School

EXAMINING COMMITTEE:

J B Kelley

D E Shipp

John A Hunter

L L Fulmer

E. A. Davis

Sam Adams

Date of Examination:

July 31, 1963